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Impact of a “Buy-online-and-pickup-in-store” Channel on Price and Quality Decisions in a Supply Chain

Xiaogang Lin, Yong-Wu Zhou, Rui Hou

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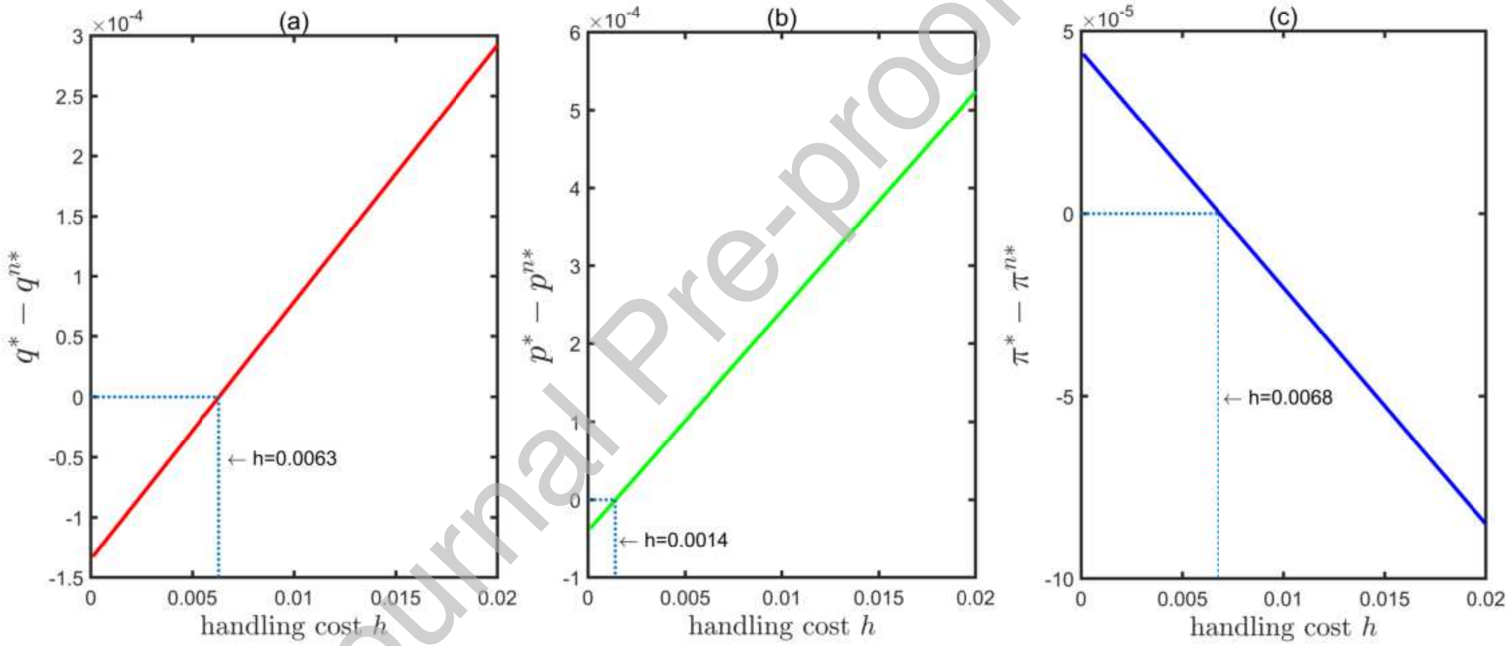
Highlights

- The impact of a "buy-online-and-pickup-in-store" channel is studied.
- Adding a "buy-online-and-pickup-in-store" channel may be profitable.
- Adding a "buy-online-and-pickup-in-store" channel might result in a lower price.
- Both manufacturer and retailer may not benefit from adding a Store channel.



(a)
(b)
(c)
$\times 10^{-4}$
$\times 10^{-4}$
$\times 10^{-5}$
$q^* - q^{n*}$
$p^* - p^{n*}$
$\pi^* - \pi^{n*}$
← h=0.0063
← h=0.0014
← h=0.0068
handling cost h
handling cost h
handling cost h

Impact of a "Buy-online-and-pickup-in-store" Channel on Price and Quality Decisions in a Supply Chain

Xiaogang Lin[†], Yong-Wu Zhou[‡], Rui Hou[†*]

[†]School of Management, Guangdong University of Technology, Guangzhou, Guangdong 510520

xglin@hotmail.com, hour@gdut.edu.cn

[‡]School of Business Administration, South China University of Technology, Guangzhou, Guangdong 510640

zyw_666@hotmail.com

Abstract

This paper studies the impact of a "buy-online-and-pickup-in-store (BOPS)" channel on quality, prices, and profits of a manufacturer and a retailer. We analyze a Stackelberg game-theoretic model where the manufacturer produces and sells a product with a quality level to the retailer at a wholesale price, and the retailer sells the product to end customers at a selling price through a Store channel, an Online channel or a BOPS channel (if available). The retailer would incur an extra handling cost if opening the BOPS channel, and customers would incur a shipping and transaction cost if purchasing from the Online channel and the Store channel, respectively. We find that both the manufacturer and the retailer can benefit from adding the BOPS channel under certain conditions. Moreover, when the BOPS channel is not available, adding the Store channel is beneficial for both parties and results in lower quality and wholesale price but higher selling price when the shipping cost is relatively high. When the Store channel is dominated by the available BOPS channel, however, opening the Store channel cannot benefit both parties. We also show that adding the BOPS channel would increase (reduce) both consumer surplus and social welfare for a sufficiently low (high) handling cost. We further observe that it is profitable for a centralized decision maker to add the BOPS channel via increasing both the price and quality under some simple conditions. Finally, we extend our base model to a more general one and illustrate our main results remain valid.

Keywords: Game theory; buy-online-and-pickup-in-store; pricing; quality; supply chain

*Corresponding author

1 Introduction

In recent years, we have witnessed the rise of "buy-online-and-pickup-in-store" strategy due to the development of e-commerce. The BOPS channel offered by retailers with brick-and-mortar stores and an online channel allows customers to pick up their online orders at stores. Compared to what customers can regularly purchase through the store channel with space constraints, the BOPS channel can help retailers offer a larger variety of products. Moreover, the channel can introduce more convenience or reduce shipping cost for those customers who purchase from the store channel or the online channel. This further can lead to channel synergy and appeal to more customers (Zhang et al. 2019b). Consequently, to manage and maximize the advantage of the online channel and the store channel, many dual-channel retailers, such as Walmart, Suning Appliance, Uniqlo, and 7 Eleven, open the BOPS channel. At the same time, since product quality can significantly affect demands for products (Xu 2009, Shi et al. 2013), manufacturers (i.e., upstream companies) need to consider how to adjust the product design and pricing when their retailers open the BOPS channel.

Despite the strengths of the BOPS channel mentioned above, there exist a number of operations challenges (Cao et al. 2016). For example, retailers would incur an extra cost associated with the drive-through service in fulfilling a customer's order if the product is available online and in-store. Also, if a product is out-of-stock or not available in-store due to space constraints, retailers need to manage an additional handling cost in order to ship the purchased product from the warehouse to the selected store for customer pickup. Moreover, the new channel might cannibalize sales from the store and online channels, and how can the manufacturers and the retailers manage this problem via product design and pricing is unclear. In fact, as mentioned in Zhang et al. (2019b), Uniqlo incurred an increased operational cost but failed to increase sales when implementing the BOPS channel in 2014. After adjustment, however, the BOPS channel offered by Uniqlo again was welcomed by customers and increased sales significantly in 2016. By observing these phenomena, we aim to answer an important and practical question of how can a manufacturer and a retailer set the quality and prices to achieve a win-win outcome, i.e., both the manufacturer and the retailer are better off, after adding the BOPS channel.

To address these issues, we employ a Stackelberg game-theoretic model to analyze a "buy-online-and-pickup-in-store (BOPS)" channel. Our objective is to study the effect of the BOPS channel on quality, prices, and profits. We consider a supply chain in which a manufacturer produces and sells a product with a quality level to the retailer at a wholesale price, and the retailer sells the product to end customers at a selling price through a store channel, an online channel or a BOPS channel (if available). The retailer would incur an extra handling cost for selling the product through the BOPS channel that might come from the coordination of online and offline information and logistics and the drive-through service for customers. Customers would incur a shipping cost and a transaction cost if purchasing from the online channel and the store channel, respectively, and they can decide to purchase the product through which channel based on their preferences. Two scenarios, i.e., the product is available for purchase only through online and both through online and in-store, are considered. Our results show that both the manufacturer and the retailer can benefit from adding the BOPS channel for not too high handling cost via decreasing the product quality and wholesale price but increasing the selling price under certain conditions. When the product is available for purchase only (both) through online (online and in-store), we indicate that adding the BOPS channel results in lower (higher) selling price for a relatively low handling cost if the shipping cost increases. Moreover, when the BOPS channel is not available, adding the Store channel is beneficial for both the manufacturer and the retailer, and they should reduce the quality and wholesale price but increase the selling price when the shipping cost is relatively high. When the Store channel is dominated by the available BOPS channel, however, opening the Store channel can not benefit both parties. In addition, we find that adding the BOPS channel would increase (reduce) both consumer surplus and social welfare for a sufficiently low (high) handling cost. We also consider the case where the decisions are made in a centralized manner and observe that it is profitable for a decision maker to add the BOPS channel via increasing both the price and quality under some simple conditions. Finally, we extend our base model to a more general one and illustrate that our main results are robust.

The remainder of this paper is organized as follows. In §2, we review related literature. In §3, we describe our model settings. In §4, we consider two scenarios where the product is available for purchase only through online and both through online and in-store, and present our analytical results. In §5, we explore two extensions. We conclude our paper in §6. All proofs are positioned in the Appendix and Supplementary materials.

2 Literature Review

Our paper adds to the considerable literature on the dual-channel supply chain. As more and more retailers and even manufacturers adopt dual channels, the scholars have paid much attention to the dual-channel supply chain problems. Most extant studies focus on pricing strategies to enable dual channel to dominate traditional retail-channel and manage the channel conflict (Tsay and Agrawal 2004b, Kumar and Ruan 2006, Bernstein et al. 2009, Chen et al. 2017). Chiang et al. (2003), Hendershott and Zhang (2006) and Chen et al. (2012) study pricing decisions for a manufacturer and an independent retailer in a dual-channel supply chain, and show that adding a direct channel can achieve a win-win outcome, i.e., both the manufacturer and the retailer earn higher profits, under certain conditions. Chen and Wang (2015) analyze the manufacturer's optimal pricing policies in free and bundled channels and the retailer's optimal subsidy policies. They highlight that power structures have a considerable impact on the decision of pricing and channel selection between a free channel and a bundled channel. Zhou et al. (2019) examine a screening model of a dual-channel supply chain in which a dominant manufacturer and a retailer engage in asymmetric operation information, and study their corresponding pricing decisions. They show that the manufacturer can extract larger profit without paying information rent under certain conditions. Modak and Kelle (2019) develop a profit maximization model to study a dual-channel supply chain under price and delivery-time dependent stochastic demand, and provide some managerial insights using a numerical example. To mitigate the channel conflict, Cattani et al. (2006) analyze a generalized equal-pricing strategy that the manufacturer commits to matching the price determined by the retailer in the traditional channel when adding a direct channel. Cai et al. (2009) investigate the impact of price discount contracts and pricing schemes on the dual-channel supply chain. They find that the price discount contracts can better manage channel conflict when the supplier enters the online direct channel. Niu et al. (2017) study the impact of channel power and fairness concern on the supplier's decision of whether to open an online direct channel, and find that these concepts can significantly affect the decision. Biswas and Avittathur (2019) propose an options contract that can coordinate a single-supplier-multiple-buyers supply chain and eliminate channel conflict. In contrast to the conventional setting in which a manufacturer and a retailer simultaneously post its wholesale price and direct price, Matsui (2017) reveals that such simultaneous price competition would not be optimal if the manufacturer and the retailer can determine the time of pricing. However, the above literature has ignored the non-price features such as quality and the impact of the rise of BOPS channel.

Closer to our research are papers considering price and service quality decisions in a dual-channel supply chain. Tsay and Agrawal (2004a) build a service competition model in a supply chain and demonstrate that adding a direct channel can benefit the supplier and the retailer. Xu (2009) and Shi et al. (2013) study price and quality decisions for a manufacturer who sells a product either through a direct channel or through a retailer. They verify that consumer heterogeneity plays a key role in the effect of different distribution channel structures on product quality. Chen et al. (2017) extend their model setting that the manufacturer can distribute a product through a retail channel, a direct channel or a dual channel with both retail and direct channels. They analyze the impacts of adding a new channel on price, quality, profit, and consumer surplus and find that adding a new channel could improve the supply chain performance. Wang et al. (2017) explore the impact of asymmetric customer loyalty on price and quality decisions for two manufacturers, and show that channel structure in the quality-sensitive market and firms' profits are affected by the customer loyalty. Zhang et al. (2019a) propose two common contracts offered by a retailer that can allow a manufacturer direct access to customers and study the retailer's contract selection and the manufacturer's quality decision. They demonstrate that the retailer can benefit from the revenue sharing (fixed fee) contract for a large (small) market heterogeneity. Zhang et al. (2019c) consider price and quantity decisions and information-sharing mechanism of a dual-channel supply chain where a manufacturer can invest in product's quality, and this investment is invisible for a retailer. They demonstrate that the retailer (the manufacturer) is more (less) willing to share the information with information leakage. Different from the above studies, our paper contributes to the literature in that we focus on the impacts of a new and growing channel called "buy-online-and-pickup-in-store" on quality, prices and profits.

Our work is also relevant to studies of the impact of a BOPS channel on pricing strategies for retailers. Cao et al. (2016) consider a retailer who sells a product to customers through Online, Store and BOPS channels and develops an analytical model to study the impact of a BOPS channel on the demand allocations, pricing, and profit. They show that adding a BOPS channel can benefit the retailer under some simple conditions. Gao and Su (2016) study the impact of the BOPS channel on store operations and demonstrate that those products sold well are not suited for in-store pickup. Jin et al. (2018) propose strategies for a retailer who adopts the BOPS channel and decides the service area. They provide guidance for determining the size of the BOPS service area and judging whether a certain type of product should be allowed for BOPS. Shi et al. (2018) study the BOPS strategy for a retailer with two classes of customers, and prove that the BOPS strategy is not necessarily beneficial to the retailer. Zhang et al. (2019b) investigate the BOPS strategy for a retailer under monopoly and competition scenarios. They find that the BOPS channel can decrease (increase) the retailer's profit in the monopoly (competition) case. Niu et al. (2019) examines the effects of the BOPS channel on traffic congestion control, and show that the BOPS channel can benefit the retailer for the high logistics cost. MacCarthy et al. (2019) study store picking operations for same day BOPS services, and obtain Best Performance Frontiers for single wave and multi-wave in-store order picking. Du et al. (2019) consider the omnichannel environment and show that consumers' anticipated disappointment aversion behavior has a significant effect on the optimal pricing decisions of retailers in the absence or presence of inventory constraint. Unlike all the above papers, our paper considers a supply chain in which the manufacturer distributes a product to a retailer who adopts Online, Store and BOPS channels. Our results highlight the impacts of the BOPS channel on not only the selling price and retailer' profit but also the quality, wholesale price and manufacturer's profit.

3 The Model

Consider a supply chain that consists of one manufacturer and one retailer. A single product with a quality level q ($q > 0$) is produced by the manufacturer and sold to the retailer at a wholesale price w ($w > 0$), where we assume that the unit production cost is a quadratic function of product quality, i.e, q^2 (Moorthy 1988, Desai 2001, Shi et al. 2013 and Chen et al. 2017). To be consistent with reality (e.g., Suning and Gome) and literature (e.g., Gao and Su 2016, Zhang et al. 2019b), we assume that the retailer sells the product at the same price p ($p > 0$) per unit to customers through the following three distribution channels: Store channel, where customers can purchase the product in a physical store; Online channel, where customers can buy the product online; and BOPS channel, where customers can purchase the product online and visit a nearby physical store to pick up it. In addition to the purchase cost paid to the manufacturer, the retailer will incur an extra handling cost h for selling the product through the BOPS channel that might come from the coordination of online and offline information and logistics and the drive-through service for customers (Cao et al. 2016, Zhang et al. 2019b).[1]

The net utility of a customer purchasing the product from the Store, Online and BOPS channels can be respectively given by

$$U_s = \theta q - p - t - l_s,\ U_o = \theta q - p - s,\ U_{bops} = \theta q - p - t, \tag{1}$$

where θ is the willingness to pay for product quality, t is the travel cost of store visits depending on the distance to the store, s (> 0) is the shipping cost when the customers purchase the product through the Online channel, l_s (> 0) is the transaction cost (e.g., spending time finding the desired items and standing in line to pay) when the customers purchase the product through the Store channel.[2] Reasonably, we allow for customer heterogeneity with respect to the willingness to pay θ and travel cost t. Following most extant literature (e.g., Villas-Boas 1998, Bhargava and Choudhary 2001, Shi et al. 2013 and Cao et al. 2016), we assume that the mass of customers in the considered market is normalized to one and each customer will purchase at most one unit of the product through one channel; and θ and t are independently and uniformly distributed over support $[0, 1]$. For brevity, we also assume that the customers will consider all available channels offered by the retailer and make a purchase from the channel with the highest utility.[3] In Table 1, we summarize the notations used in the paper.

[1]One may argue that the retailer would incur an extra cost for selling the product through the Store channel compared with the Online channel or BOPS channel due to the extra storage/insurance for keeping the product in the store. We will show that our results still hold if we incorporate the cost to the base model in a later extension.

[2]Note that in the following profit function of the retailer, the revenue from charging customers a shipping cost is ignored because we assume that the retailer will pass the shipping cost on to the customers. Furthermore, we implicitly assume that the transaction cost when the customers purchase the product through the Online or BOPS channel is negligible because purchasing the product online is just "a click away" compared with the store transaction cost (Zhang et al. 2019b). We will relax this assumption and show that our main results remain valid in a later extension.

[3]In practice, some customers who are old-fashioned and unwilling to shop online or do not have Internet access might prefer the Store channel, regardless of whether the Online and/or BOPS channels are available. We will categorize the customers into multiple segments and show that our results remain valid in a later extension. Also, one might argue that some customers (typically "millennial" customers) might prefer to order online (i.e., purchase through the Online channel or the BOPS channel), regardless of whether the Store channel is available. We, however, will explain the reason why our results would not change if we do not incorporate this customer segment later.

Table 1: Summary of mathematical notations

q = quality level of the product;
w = wholesale price of the product;
p = selling price of the product;
θ = willingness to pay for product quality;
h = handling cost incurred by the retailer for a purchase through the BOPS channel;
t = travel cost incurred by the customers for a purchase through the Store channel;
l_s = transaction cost incurred by the customers for a purchase through the Store channel;
s = shipping cost incurred by the customers for a purchase through the Online channel.

The timeline of our model is as follows. The retailer first announces whether or not to open the BOPS channel. Depending on the decision, the manufacturer first determines the quality q and then the wholesale price w. Afterward, the retailer determines the price p charged to end customers so as to maximize the profit.

4 Results

We solve the game in a backward fashion. Given any quality and wholesale price quoted by the manufacturer, the retailer determines the selling price for the product. Anticipating the retailer's optimal pricing decision, the manufacturer determines the wholesale price and quality. To analyze the impacts of the BOPS channel, we investigate the following scenarios:

- The product is available for purchase only through online. The reason why we consider this case is that some retailers can only sell the product online due to constraints for display on the physical size of retail stores. Under this scenario, customers can order the product online and have the product shipped to a specified address (i.e., purchase through the Online channel); or purchase the product online but pick up it in a nearby store if the BOPS channel is offered by the retailer.

- The product is available for purchase through online or in-store. Under this scenario, customers can order the product through the Online or Store channel, or purchase the product online but pick up it in a nearby store if the BOPS channel is offered by the retailer.

In this paper, we assume that $0 < l_s \leq s \leq 1$, $s < \overline{s}$, and $0 < h < \frac{1-8s-8s^2+\sqrt{1-8s^2}}{8s}$, where $\overline{s}$ is the unique positive solution of the equation $8s^3 + 16s^2 + 7s - 2 = 0$. These assumptions are not restrictive, and the reason why we employ them are to ensure: (1) positive customer demand for the Store channel given any quality q and selling price p (i.e., $q - p > l_s$); (2) positive customer demand for the Online channel given any quality q and selling price p (i.e., $q - p > s$); (3) the Online channel is not always dominated by the Store channel, and vice versa (i.e., $0 \leq s - l_s \leq 1$); (4) the Online channel is not always dominated by the BOPS channel (i.e., $s \leq 1$). Note that we can rewrite (3) and (4) as $0 < l_s \leq s \leq 1$. Furthermore, after deriving optimal solutions of q and p under the above scenarios in the appendix, we conclude that (1) and (2) become $s < \overline{s}$ and $0 < h < \frac{1-8s-8s^2+\sqrt{1-8s^2}}{8s}$.[4]

4.1 Product Available Online Only

4.1.1 BOPS Channel Not Offered

In this scenario, the BOPS channel is not available. In other words, customers can purchase the product only through the Online channel. We assume that they will purchase the product if and only if $U_o \geq 0$. Denote by D_o^{on} the customers' demand for the Online channel with the product available online only in the absence of the BOPS channel. We can characterize their purchasing behavior in Figure 1(i), where line (a) is the indifference curve between no-buy and purchase from the Online channel. Clearly, the demand for the Online channel can be given by $D_o^{on} = \frac{q-p-s}{q}$. Therefore, the profit functions of the retailer and the manufacturer are respectively given by

$$\pi_r^{on} = (p - w)D_o^{on} \text{ and } \pi_m^{on} = (w - q^2)D_o^{on}.$$

Moreover, consumer surplus CS^{on} and social welfare SW^{on} are as follows:

$$\begin{cases} CS^{on} = \int_{\frac{p+s}{q}}^{1} \int_0^1 (\theta q - p - s)dtd\theta = \frac{(q-p-s)^2}{2q}, \\ SW^{on} = \pi_r^{on} + \pi_m^{on} + CS^{on}. \end{cases} \tag{2}$$

Consequently, we can easily obtain the following lemma.

[4] Note that the assumption can be relaxed under some scenarios. For example, when the product is available online only and the BOPS channel is not offered, (1) and (2) can be rewritten as $s < \frac{1}{4}$ (see the proof of Lemma 1 in the Appendix). For consistency, we make the strongest assumption in our decentralized model, this however would not change our main results.

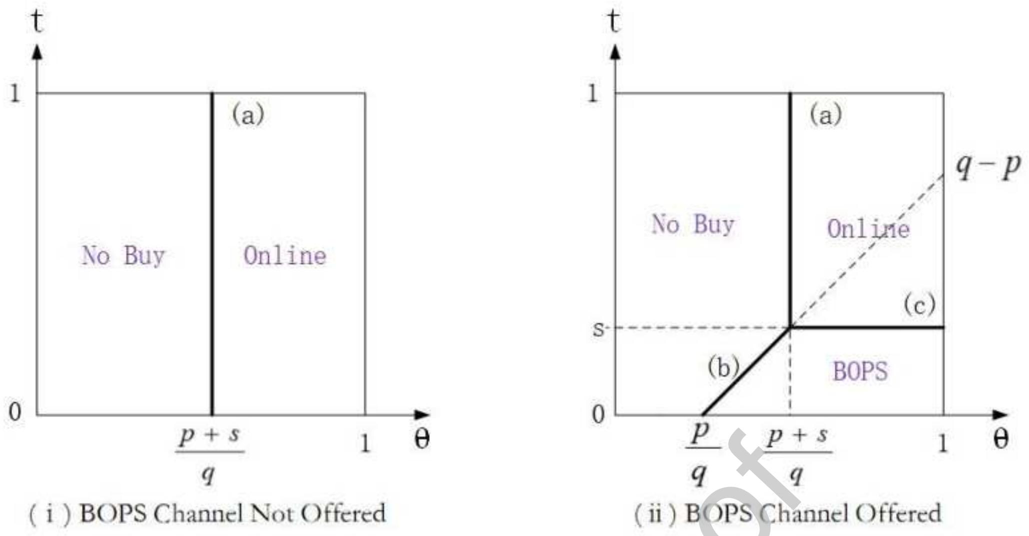


Figure 1: Purchasing options for a product offered Online only

Lemma 1. *If the BOPS channel is not offered by the retailer, the retailer's optimal selling price and the manufacturer's optimal wholesale price and quality are as follows:* $p^{on*} = \frac{5-24s+5\sqrt{1+12s}}{36}$, $w^{on*} = \frac{1-3s+\sqrt{1+12s}}{9}$, *and* $q^{on*} = \frac{1+\sqrt{1+12s}}{6}$.

By Lemma 1, we can analyze the impact of the shipping cost s on the optimal solutions and obtain the following proposition.

Proposition 1. *When s increases, the optimal wholesale price and quality increase, the optimal profits of the manufacturer and the retailer decrease, but the optimal selling price increases for $s \leq \frac{3}{64}$, and decreases otherwise.*

Proposition 1 reveals that the manufacturer should increase the quality and thereby the wholesale price (to cover the production cost) to appeal to customers if they incur a high shipping cost. Interestingly, the retailer, however, can increase the selling price for a relatively low shipping cost. Note that this stands in contrast to the result obtained by Cao et al. (2016), who show that the selling price always decreases in s. The intuition is that the retailer can increase the price to bear the increased wholesale price and make profit because the manufacturer can appeal to customers via improving the quality for low shipping cost. But when the shipping cost is high enough, the retailer should also reduce the selling price to appeal to customers even if the quality has been improved. Since the demand decreases in the shipping cost (see Appendix), it is intuitive that both the manufacturer and the retailer's profits decrease in the shipping cost.

4.1.2 BOPS Channel Offered

We now consider that the retailer opens the BOPS channel, i.e., customers can purchase the product through the Online or BOPS channel, based on their preference. They will purchase the product through the i channel if and only if $U_i \geq 0$ and $U_i \geq U_j$, where $i, j = o, bops$ and $i \neq j$. Denote by D_o^o and D_{bops}^o the customers' demands for the Online channel and the BOPS channel with the product online only in the presence of the BOPS channel, respectively. From Figure 1(ii), where line (a) is the indifference curve between no-buy and purchase from the Online channel, and line (b) is the indifference curve between no-buy and purchase from the BOPS channel, and line (c) is the indifference curve between purchase from the Online and the BOPS channel.[5] We thus obtain $D_o^o = \frac{(q-p-s)(1-s)}{q}$ and $D_{bops}^o = \frac{(2q-2p-s)s}{2q}$. Therefore, the profit functions of the retailer and the manufacturer are respectively given by

$$\pi_r^o = (p-w)D_o^o + (p-w-h)D_{bops}^o \text{ and } \pi_m^o = (w-q^2)(D_o^o + D_{bops}^o).$$

Furthermore, consumer surplus CS^o and social welfare SW^o are respectively given by:

$$\begin{cases} CS^o = \int_{\frac{p+s}{q}}^1 \int_s^1 (\theta q - p - s) dt d\theta + \int_{\frac{p}{q}+\frac{1-\frac{p}{q}}{q-p}t}^1 \int_0^s (\theta q - p - t) dt d\theta = \frac{3(q-p-s)^2 + s^2[3(q-p)-2s]}{6q}, \\ SW^o = \pi_r^o + \pi_m^o + CS^o. \end{cases} \quad (3)$$

Consequently, we can obtain the following lemma.

Lemma 2. *If the BOPS channel is offered by the retailer, the retailer's optimal selling price and the manufacturer's optimal wholesale price and quality are as follows:* $p^{o*} = \frac{5-12s(2-h-s)+5\sqrt{1+6s(2+2h-s)}}{36}$, $w^{o*} = \frac{2-3s(2+2h-s)+2\sqrt{1+6s(2+2h-s)}}{18}$, *and* $q^{o*} = \frac{1+\sqrt{1+6s(2+2h-s)}}{6}$.

By Lemma 2, we also can analyze the impacts of the shipping cost s and the handling cost h on the optimal solutions, respectively.

[5] Note that the ordinate $q-p$ of line (c) at the origin in Figure 1(ii) can be larger than one. However, the demand functions for the Online and BOPS channel remain consistent.

Proposition 2. *(a) When s increases, the optimal wholesale price and quality increase, the optimal profits of the manufacturer and the retailer decrease, but the optimal selling price increases for $s \leq \overline{s}_0$, and decreases otherwise, where $\overline{s}_0 > \frac{3}{64}$ is increasing in h.*

(b) When h increases, the optimal selling price, wholesale price, and quality increase, while the optimal profits of the manufacturer and the retailer decrease.

Proposition 2(a) also indicates that when the shipping cost becomes higher, the manufacturer should increase the quality and the wholesale price to guarantee demands for the Online and BOPS channels, and the retailer can increase the selling price for a relatively low shipping cost. And likewise, when the customers incur a higher shipping cost, both the manufacturer and the retailer's profits decrease because the demand for the Online channel decreases (although the demand for the BOPS channel increases) (see Appendix). However, it is worthy to note that if the retailer offers the BOPS channel, the threshold such that the selling price increases in s is higher than that of the case in which the retailer offers only the Online channel. In other words, the retailer who opens the BOPS channel can increase the price for a relatively higher shipping cost. The intuition is that when the shipping cost increases, more customers will switch to the BOPS channel. The retailer thus can increase the price to bear the increased wholesale price and still make profit for a not too high shipping cost (note the manufacturer can appeal to customers via improving the quality). Reasonably, Proposition 2(b) verifies that the retailer will increase the selling price when the retailer incurs a higher handling cost for a purchase through the BOPS channel. To appeal to customers, the manufacturer will improve quality and thereby increase the wholesale price. This makes the demands for the Online and BOPS channels decrease (see Appendix) and thereby leads to lower profits of the manufacturer and the retailer.

4.1.3 Impact of the BOPS Channel

According to Lemma 1 and Lemma 2, we now can investigate the impact of adding the BOPS channel from the perspectives of profitabilities, quality, and prices when the product is available for purchase only through online. We first compare the quality and prices in the absence and the presence of the BOPS channel, which is characterized in the following theorem.

Theorem 1. *If the retailer offers the BOPS channel, there exist thresholds $\overline{h}_m$ and $\overline{h}_p$ such that the manufacturer should decrease the quality and the wholesale price if and only if $h \leq \overline{h}_m$, and the* *retailer should decrease the selling price if and only if $h \leq \overline{h}_p$ compared with the case in which the retailer does not offer the BOPS channel. Moreover, $\overline{h}_p < \overline{h}_m$, where $\overline{h}_m$ increases in s while $\overline{h}_p$ first increases then decreases in s.*

Theorem 1 tells that compared with the case in which the BOPS channel is not available, when the handling cost is not too high (i.e., $h \leq \overline{h}_m$), the manufacturer will help the retailer who adds the BOPS channel handle the cost better through lower wholesale price with low product quality. When the cost is high, in order to help the retailer address the cost, however, the manufacturer will improve the quality and appeal to more customers but charge a higher wholesale price. Moreover, the manufacturer will reduce the wholesale price by producing the product with low quality for a higher handling cost if the shipping cost increases. The reason is that higher shipping cost will make customers switch to the BOPS channel, which implies that the retailer who adds the BOPS channel incurs higher handling cost, the manufacturer thus will employ low wholesale price strategy to help the retailer handling the higher cost. Therefore, when the handling cost is sufficiently low (i.e., $h \leq \overline{h}_p < \overline{h}_m$), the retailer should decrease the selling price and attract more customers due to low product quality. And when the cost is relatively high (i.e., $\overline{h}_p < h \leq \overline{h}_m$), the retailer needs to increase the price to bear the high handling cost. Moreover, when the shipping cost is relatively low (high), there are not too many (many) customers switching from the Online channel to the BOPS channel, which implies that the retailer does not (does) bear high total handling cost. Consequently, if the shipping cost increases, which makes more customers choose the BOPS channel, the retailer can reduce the price for a higher (lower) handling cost. We note that the price change even in a (simple) supply chain setting is quite different from the result obtained by Cao et al. (2016), who do not consider a manufacturer producing and selling the product to the retailer and show that the selling price always increases when the retailer adds the BOPS channel.

We then compare the profits of the retailer and the manufacturer in the absence and the presence of the BOPS channel.

Theorem 2. *It is profitable for the retailer to add the BOPS channel if and only if $h \leq \overline{h}$, where $\overline{h}$ first increases and then decreases in s. Moreover, the manufacturer's profit increases when the retailer opens the BOPS channel.*

Theorem 2 shows that if the handling cost is relatively low, a win-win outcome can be achieved, i.e., both the retailer and the manufacturer benefit from adding the BOPS channel. This result is reasonable as the handling cost is borne by the retailer and might pass to the manufacturer, and they cannot better manage a too high handling cost that would lead to low profits through quality and prices. In addition, when the shipping cost is relatively low (high), a win-win outcome can be reached for a higher (lower) handling cost if the shipping cost increases. As mentioned, the retailer does not (does) bear a high total handling cost as the shipping cost is relatively low (high). Consequently, if the shipping cost increases, the retailer and the manufacturer can manage a higher (lower) handling cost and benefit from adding the BOPS channel.

Combined with Theorem 1 and Theorem 2, we can analyze the change in quality and prices when it is profitable for the retailer to add the BOPS channel compared with the case in which the retailer does not offer the BOPS channel.

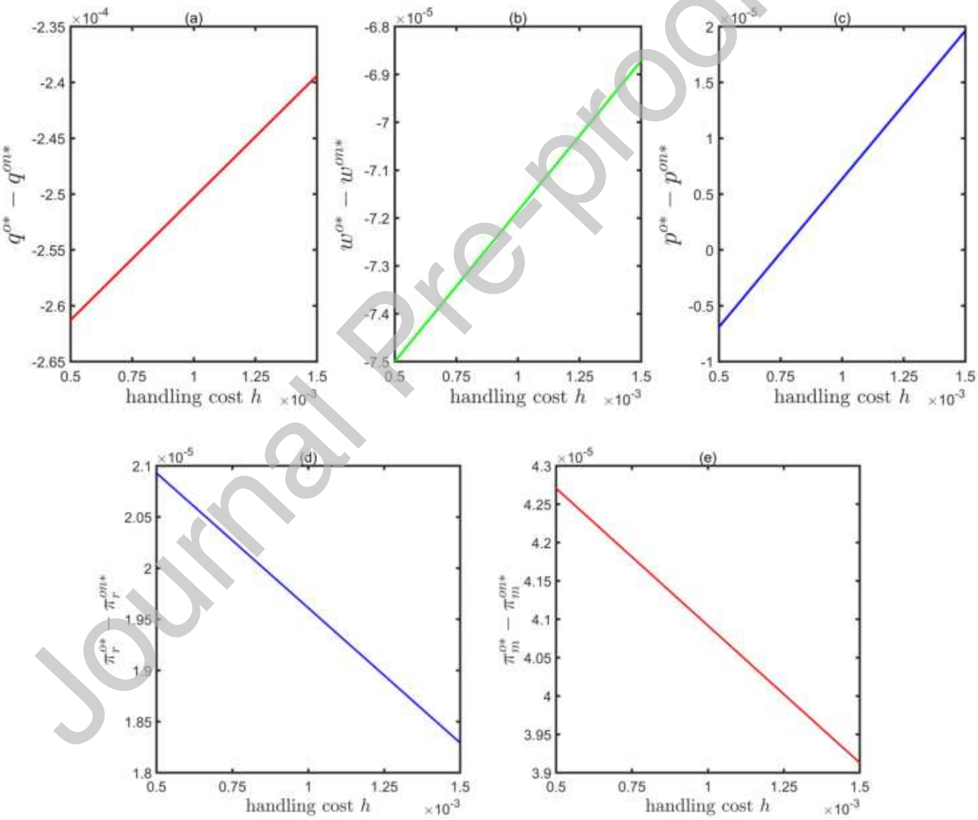


Figure 2: Impacts of adding the BOPS channel for $h \leq \overline{h}$ when the product is available online only

Corollary 1. *When the retailer chooses to add the BOPS channel (i.e., $h \leq \overline{h}$ such that $\pi_r^{o*} \geq \pi_r^{on*}$*

and $\pi_m^{o*} \geq \pi_m^{on*}$), the manufacturer should lower the quality and wholesale price. However, the retailer should reduce the selling price for relatively low handling cost (i.e, $h \leq \overline{h}_p$), but increase the selling price for mediate handling cost (i.e, $\overline{h}_p < h \leq \overline{h}$).

In order to achieve a win-win outcome if the retailer chooses to add the BOPS channel (see Figures 2(d)-(e)),[6] Corollary 1 reveals that the manufacturer need to help the retailer who adds the BOPS channel manage the handling cost through low wholesale price with low product quality (see Figures 2(a)-(b)) as the handling cost is not too high. Moreover, as explained in Theorems 1 and 2, the retailer should decrease the price for relatively low handling cost while increase the price for mediate handling cost (see Figure 2(c)).

Conventional wisdom has it that adding the BOPS channel can always improve both consumer surplus and social welfare because consumers have an additional purchasing channel. However, as shown in the following corollary, this is not the case.

Corollary 2. *If the handling cost h is sufficiently low (high), adding the BOPS channel would improve (harm) consumer surplus and social welfare.*

Corollary 2 confirms that adding the BOPS channel might not benefit consumers and improve social welfare (for a sufficiently high handling cost). The intuition is that a high handling cost would lead to a high selling price charged to customers, although the product quality might increase (see Theorem 1). This in turn makes consumers unwilling to purchase the product even if there exists another purchasing channel (i.e., BOPS channel). If the handling cost is sufficiently low, however, both consumer surplus and social welfare would increase because the selling price reduces and thereby the BOPS channel indeed helps increase customer demand.

4.2 Product Available Both Online and In-Store

4.2.1 BOPS Channel Not Offered

In this scenario, customers can purchase the product only through the Online or the Store channel, based on their preference. They will purchase the product through the i channel if and only if $U_i \geq 0$ and $U_i \geq U_j$, where $i, j = o, s$ and $i \neq j$. Denote by D_o^n and D_s^n the customers' demands for the Online channel and the Store channel with the product both online and in-store in the absence

[6]Note $s = 0.0249$ and $h \in \{0.0005, 0.00055, \ldots, 0.0015\}$.

of the BOPS channel, respectively. We can characterize their purchasing behavior in Figure 3(i), where line (a) is the indifference curve between no-buy and purchase from the Online channel, and line (b) is the indifference curve between no-buy and purchase from the Store channel, and line (c) is the indifference curve between purchase from the Online and the Store channels.[7] We thus

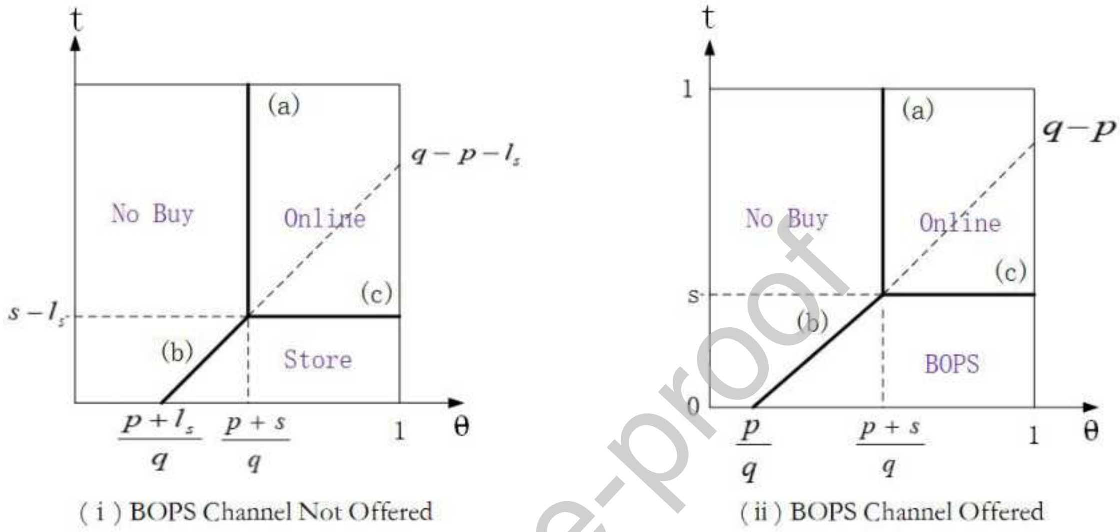


Figure 3: Purchasing options for a product offered both Online and In-store

obtain $D_o^n = \frac{(q-p-s)(1-s+l_s)}{q}$ and $D_s^n = \frac{(2q-2p-s-l_s)(s-l_s)}{2q}$. Therefore, the profit functions of the retailer and the manufacturer are respectively given by

$$\pi_r^n = (p-w)(D_o^n + D_s^n) \text{ and } \pi_m^n = (w-q^2)(D_o^n + D_s^n).$$

Moreover, consumer surplus CS^n and social welfare SW^n are respectively given by:

$$\begin{cases} CS^n = \int_{\frac{p+s}{q}}^{1} \int_{s-l_s}^{1} (\theta q - p - s) dt d\theta + \int_{\frac{p+l_s}{q} + \frac{1-\frac{p+l_s}{q}}{q-p-l_s} t}^{1} \int_{0}^{s-l_s} (\theta q - p - t - l_s) dt d\theta \\ \quad = \frac{3(q-p-s)^2 + s^2[3(q-p)-2s]}{6q} - \frac{l_s[l_s^2 - 3l_s(q-p) + 3s(2q-2p-s)]}{6q}, \\ SW^n = \pi_r^n + \pi_m^n + CS^n. \end{cases} \tag{4}$$

Let $\phi = s^2 - 2(1+l_s)s + l_s^2 < 0$, we then can obtain the following lemma.

[7] Note that the ordinate $q-p-l_s$ of line (c) at the origin in Figure 3(i) can be larger than one. However, the demand functions for the Online and Store channel remain consistent.

Lemma 3. *If the BOPS channel is not offered by the retailer, the retailer's optimal selling price and the manufacturer's optimal wholesale price and quality are as follows:* $p^{n*} = \frac{5+12\phi+5\sqrt{1-6\phi}}{36}$, $w^{n*} = \frac{2+3\phi+2\sqrt{1-6\phi}}{18}$ *and* $q^{n*} = \frac{1+\sqrt{1-6\phi}}{6}$.

By Lemma 3, we can analyze the impacts of the shipping cost s and the transaction cost l_s on the optimal solutions.

Proposition 3. *(a) When s or l_s increases, the optimal wholesale price and quality increase, but the optimal profits of the manufacturer and the retailer decrease.*

(b) When s increases, if $l_s \geq \frac{3}{64}$, the optimal selling price always decreases, but if $l_s < \frac{3}{64}$, the optimal selling price increases for $s \leq \widetilde{s}_0$, and decreases otherwise, where $\widetilde{s}_0$ is decreasing in l_s.

(c) When l_s increases, if $s \leq \frac{3}{64}$, the optimal selling price increases, if $s \geq 1 - \frac{\sqrt{58}}{8}$, the optimal selling price decreases, but if $\frac{3}{64} < s < 1 - \frac{\sqrt{58}}{8}$, the optimal selling price increases for $l_s \leq \widetilde{l}_0$, and decreases otherwise, where $\widetilde{l}_0$ is decreasing in s.

Proposition 3 shows that when the shipping cost or the transaction cost becomes higher, the total demand for the Online and the Store channels decrease (see Appendix), which reduces the manufacturer and the retailer's profits. To guarantee demands for the Online and the Store channels, the manufacturer thus should increase the quality and thereby the wholesale price to cover production cost. However, we note that the optimal selling price is not necessarily monotonic in the shipping cost or the transaction cost. The intuition is that when the shipping cost or the transaction cost increases, more customers will switch to the Store or the Online channel. The retailer thus can increase or decrease the price to bear the purchasing cost and make profit.

4.2.2 BOPS Channel Offered

We now consider that the retailer opens the BOPS channel, i.e., customers can purchase the product through the Online or Store or BOPS channel, based on their preference. They will purchase the product through the i channel if and only if $U_i \geq 0$ and $U_i \geq \max\{U_j, j = o, s, bops\}$, where $i = o, s, bops$ and $i \neq j$. From (1), one can easily know that the Store channel is dominated by the

BOPS channel, i.e., $U_s \leq U_{bops}$.[8] In other words, customers will choose between the Online and the BOPS channels, which is consistent with Section 4.1.2. This implies that the optimal solutions are the same as that of Section 4.1.2. We then can obtain the following results.

Lemma 4. *If the BOPS channel is offered by the retailer, the retailer's optimal selling price and the manufacturer's optimal wholesale price and quality are as follows:* $p^* = \frac{5-12s(2-h-s)+5\sqrt{1+6s(2+2h-s)}}{36}$, $w^* = \frac{2-3s(2+2h-s)+2\sqrt{1+6s(2+2h-s)}}{18}$, *and* $q^* = \frac{1+\sqrt{1+6s(2+2h-s)}}{6}$.

Proposition 4. *(a) When s increases, the optimal wholesale price and quality increase, the optimal profits of the manufacturer and the retailer decrease, but the optimal selling price increases for $s \leq \bar{s}_0$, and decreases otherwise, where $\bar{s}_0$ is increasing in h.*

(b) When h increases, the optimal selling price, wholesale price, and quality increase, while the optimal profits of the manufacturer and the retailer decrease.

4.2.3 Impact of the BOPS Channel

From Lemma 3 and Lemma 4, we now analyze the impact of adding the BOPS channel on the profit, quality, and prices as the product is available both online and in-store, We first compare the quality and prices in the absence and the presence of the BOPS channel, which is characterized in the following theorem.

Theorem 3. *If the retailer offers the BOPS channel, there exist thresholds $\widetilde{h}_m$ and $\widetilde{h}_p$ such that the manufacturer should decrease the quality and the wholesale price if and only if $h \leq \widetilde{h}_m$, and the retailer should decrease the selling price if and only if $h \leq \widetilde{h}_p$ compared with the case in which the retailer does not offer the BOPS channel. Moreover, $\widetilde{h}_p < \widetilde{h}_m$, where $\widetilde{h}_m$ increases in s and l_s while $\widetilde{h}_p$ increases in l_s but decreases in s.*

Similar to what happened in the scenario that the product is available online only, Theorem 3 reveals that if the product is available both online and in-store, when the handling cost is not too

[8]This setting is consistent with that of Cao et al. (2016) and Zhang et al. (2019b). We, however, will consider the case in which there exists a number of customers preferring the Store channel, regardless of whether the Online and/or BOPS channels are available in a later extension. Moreover, note that we do not need to consider those customers who prefer to order online (i.e., purchase through the Online or BOPS channel) because the demand for the channel is always non-zero.

high (i.e., $h \leq \widetilde{h}_m$), the manufacturer will help the retailer who adds the BOPS channel handle the cost better through lower wholesale price with low product quality. When the cost is high, in order to help the retailer address the cost, however, the manufacturer will improve the quality and appeal to more customers but charge a higher wholesale price. Moreover, the manufacturer will reduce the wholesale price by producing the product with low quality for a higher handling cost if the shipping cost or the transaction cost increases. The reason is that higher shipping cost or transaction cost will make customers switch to the BOPS channel, which implies that the retailer who adds the BOPS channel incurs higher handling cost, the manufacturer thus will employ low wholesale price strategy to help the retailer handling the higher cost. Therefore, when the handling cost is sufficiently low (i.e., $h \leq \widetilde{h}_p < \widetilde{h}_m$), the retailer should decrease the selling price and attract more customers due to low product quality. And when the cost is relatively high (i.e., $\widetilde{h}_p < h \leq \widetilde{h}_m$), the retailer needs to increase the price to bear the high handling cost.

However, if the shipping cost increases, the retailer should reduce the price for a lower handling cost. The reason is that more customers will switch to the Store channel when the retailer does not offer the BOPS channel and the shipping cost increases, this can help the retailer make profit without bearing the handling cost compared with the case in which more customers choose the BOPS channel but the retailer needs to manage higher handling cost. Clearly, this statement implies that the retailer can reduce the price for a higher handling cost if the transaction cost incurred by customers who purchase the product from the Store channel becomes higher.

We then compare the profits of the retailer and the manufacturer in the absence and the presence of the BOPS channel.

Theorem 4. *It is profitable for the retailer to add the BOPS channel if and only if $h \leq \widetilde{h}$, where $\widetilde{h}$ increases in l_s but decreases in s. Moreover, the manufacturer's profit increases when the retailer opens the BOPS channel.*

Theorem 4 also shows that if the handling cost is relatively low, a win-win outcome can be achieved, i.e., both the retailer and the manufacturer benefit from adding the BOPS channel. In contrast to Theorem 2, a win-win outcome can be reached for a lower and higher handling cost if the shipping cost and the transaction cost increases, respectively. As mentioned, if the shipping cost or the transaction cost increases, the retailer and the manufacturer can manage a lower or higher handling cost and benefit from adding the BOPS channel.

Combined with Theorem 3 and Theorem 4, we can analyze the change in quality and prices when it is profitable for the retailer to add the BOPS channel compared with the case in which the retailer does not offer the BOPS channel.

Corollary 3. *When the retailer chooses to add the BOPS channel (i.e., $h \leq \widetilde{h}$ such that $\pi_r^* \geq \pi_r^{n*}$ and $\pi_m^* \geq \pi_m^{n*}$), the manufacturer should lower the quality and wholesale price. However, the retailer should reduce the selling price for relatively low handling cost (i.e, $h \leq \widetilde{h}_p$), but increase the selling price for mediate handling cost (i.e, $\widetilde{h}_p < h \leq \widetilde{h}$).*

In order to achieve a win-win outcome if the retailer chooses to add the BOPS channel, Corollary 3 also verifies that the manufacturer needs to help the retailer who adds the BOPS channel manage the handling cost through low wholesale price with low product quality as the handling cost is not too high; and the retailer should decrease the price for relatively low handling cost while increasing the price for mediate handling cost.

Similar to the case in which the product is available online only (Section 4.1), we also confirm that when the product is available both online and in-store, consumer surplus and social welfare do not necessarily improve for adding the BOPS channel. Specifically, we have the following result and similar explanation can be applied as in Corollary 2.

Corollary 4. *Adding the BOPS channel would improve (harm) consumer surplus and social welfare for a sufficiently low (high) handling cost h.*

4.3 Impact of the Store channel

In this section, we compare our results in the following two scenarios: (i) The product is available online only, and (ii) the product is sold through online and in-store. We aim at answering the question of whether the retailer should open the Store channel and how the Store channel affects the quality and pricing decisions of the manufacturer and retailer when the BOPS channel is not offered or offered, respectively.

4.3.1 BOPS Channel Not Offered

We have known that when the cost of handling pickup orders is relatively high, the retailer would not open the BOPS channel due to the reduced profit. Under this scenario, one may ask whether the retailer should open the Store channel. To answer this question, from Lemma 1 and Lemma 3, we have the following corollary.

Corollary 5. *If the BOPS channel is not offered by the retailer, we have* $q^{n*} \le q^{on*}$, $w^{n*} \le w^{on*}$, $\pi_r^{n*} \ge \pi_r^{on*}$, *and* $\pi_m^{n*} \ge \pi_m^{on*}$, *Moreover, there exists a threshold* s' *such that* $p^{n*} \le p^{on*}$ *for* $s \le s'$, *and* $p^{n*} > p^{on*}$ *for* $s > s'$, *where* $\frac{3}{64} < s' < \min\{\frac{1}{4}, 1 + l_s - \frac{\sqrt{58+128l_s}}{8}\}$.

Corollary 5 tells us that when the BOPS channel is not offered by the retailer, the retailer and the manufacturer can achieve a win-win outcome, i.e., both of their profits increase, via opening the Store channel. The intuition is that the retailer who opens the Store channel can appeal to more customers to purchase the product (see Figures 1(i) and 3(i) for illustration). Specifically, compared to the case in which the retailer opens the Online channel only, the potential increased demand due to the existence of the Store channel would make the manufacturer reduce the quality and thereby the wholesale price of the product. The retailer, however, may increase the price when the shipping cost incurred by the customers who purchase via the Online channel is relatively high because customers are more willing to purchase via the Store channel. We conclude that if the BOPS channel is not offered by the retailer, both the manufacturer and the retailer should make the Store channel available because it would bring them higher profits.

4.3.2 BOPS Channel Offered

When the BOPS channel is offered by the retailer, according to (1), one can easily know that the Store channel is always dominated by the BOPS channel, which is also consistent with Cao et al. (2016) and Zhang et al. (2019b). The reason is that the customers who purchase via the BOPS channel need not to pay a transaction cost (e.g., spending time finding the desired items and stand in line to pay). No matter whether the product is available online only or sold through online and in-store, therefore, there always exists no demand for the Store channel. From Lemma 2 and Lemma 4, we obtain the following corollary.

Corollary 6. *If the BOPS channel is offered by the retailer, we have* $p^{o*} = p^*$, $w^{o*} = w^*$, $q^{o*} = q^*$, $\pi_r^{o*} = \pi_r^*$ *and* $\pi_m^{o*} = \pi_m^*$.

By Corollary 6, when the BOPS channel is offered by the retailer, the quality and pricing decisions and the corresponding profits are essentially identical whether the Store channel is available or not. In other words, when the BOPS channel is available, the retailer can shut down the Store channel (and thereby reduce operating cost) because it would not bring extra revenue.

5 Extensions

5.1 Decisions Are Made in a Centralized Manner

In our base model, we assume that the manufacturer and the retailer are independent decision makers. In practice, however, they may merge with each other and collude on the price and quality, or the manufacturer might directly sell the product to end customers. In the following, we investigate the scenario that the decisions are made in a centralized manner, i.e., the decision maker determines the selling price and the quality. The timeline of this model is that the decision maker first determines whether or not to open the BOPS channel. Depending on the decision, the maker first determines the quality and then the selling price charged to end customers so as to maximize the profit. We aim at investigating the impact of adding the BOPS channel on the profit, price, and quality.

We also consider two scenarios, i.e., the product is available for purchase only through online; and the product is available for purchase through both online and in-store. First, if the product is available only online, when the BOPS channel is not offered and offered by the decision maker, the profit functions are respectively given by

$$\pi^{on} = (p - q^2)D_o^{on} \text{ and } \pi^o = (p - q^2)D_o^o + (p - q^2 - h)D_{bops}^o,$$

where $D_o^{on} = \frac{q-p-s}{q}$, $D_o^o = \frac{(q-p-s)(1-s)}{q}$ and $D_{bops}^o = \frac{(2q-2p-s)s}{2q}$. Second, if the product is available both online and in-store, when the BOPS channel is not offered and offered by the decision maker, the profit functions are respectively given by

$$\pi^n = (p - q^2)(D_o^n + D_s^n) \text{ and } \pi = (p - q^2)D_o^o + (p - q^2 - h)D_{bops}^o,$$

where $D_o^n = \frac{(q-p-s)(1-s+l_s)}{q}$ and $D_s^n = \frac{(2q-2p-s-l_s)(s-l_s)}{2q}$. Following the same procedure as in the base model, we can obtain the following lemma.[9]

[9]For brevity, we omit the proof, which is available from the authors upon request.

Lemma 5. *(i) If the product is available for purchase only through online, the optimal solutions are as follows:*

(a) When the BOPS channel is not offered, $p^{on*} = \frac{1-3s+\sqrt{1+12s}}{9}$, $q^{on*} = \frac{1+\sqrt{1+12s}}{6}$ *and* $\pi^{on*} = \frac{(1-12s+\sqrt{1+12s})^2}{54(1+\sqrt{1+12s})}$.

(b) When the BOPS channel is offered, q^{o*} *is the unique solution of* $3q^4 - 4q^3 + [1+2(1+h)s - s^2]q^2 - [\frac{s^2}{4} - (1-h)s + 1 + h^2]s^2 = 0$ *such that the profit is maximal, and* $p^{o*} = \frac{s^2-2(1-h)s+2q^{o*}(1+q^{o*})}{4}$.

(ii) If the product is available for purchase both through online and in-store, the optimal solutions are as follows:

(a) When the BOPS channel is not offered, $p^{n*} = \frac{2+3\phi+2\sqrt{1-6\phi}}{18}$, $q^{n*} = \frac{1+\sqrt{1-6\phi}}{6}$ *and* $\pi^{n*} = \frac{(1+6\phi+\sqrt{1-6\phi})^2}{54(1+\sqrt{1-6\phi})}$, *where* $\phi = s^2 - 2(1+l_s)s + l_s^2 < 0$.

(b) When the BOPS channel is offered, q^* *is the unique solution of* $3q^4 - 4q^3 + [1+2(1+h)s - s^2]q^2 - [\frac{s^2}{4} - (1-h)s + 1 + h^2]s^2 = 0$ *such that the profit is maximal, and* $p^* = \frac{s^2-2(1-h)s+2q^{o*}(1+q^{o*})}{4}$.

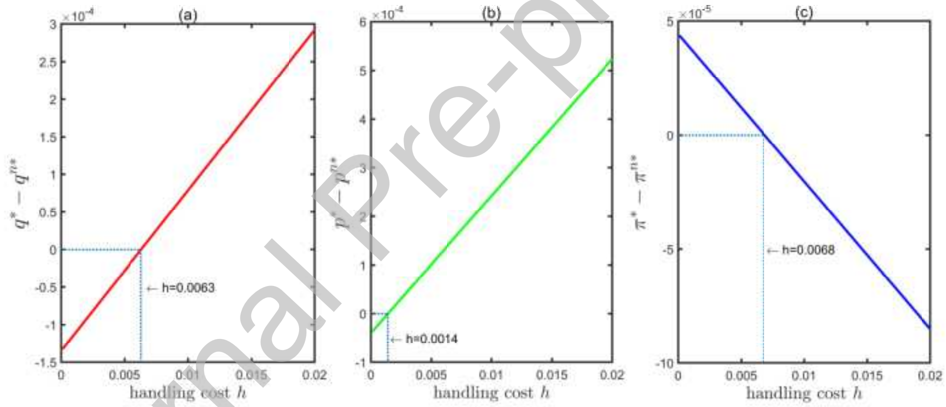


Figure 4: Impacts of adding the BOPS channel when the product is available both online and in-store and the decisions are made in a centralized manner

Unfortunately, the impacts of adding the BOPS channel on the profit, price, and quality cannot be proved analytically, we thus turn to numerical studies, and vary s, h and l_s from 0 to 0.025, respectively. As depicted in Figure 4 (note that one can obtain similar results when the product is available online only),[10] our numerical results also show that when the handling cost is not too high, adding the BOPS channel is profitable for the decision maker, no matter whether the product

[10]Note s=0.02, l_s=0.01, and $h \in \{0.0001, 0.0002, \ldots, 0.02\}$.

is available only online or both online and in-store. Moreover, the decision maker can make more profit for adding the BOPS channel through decreasing the quality and price for relatively low handling cost, and decreasing the quality but increasing the price for mediate handling cost, and finally increasing the quality and price for not too high handling cost. The reason is as follows. As depicted in Figures 1 and 3, adding the BOPS channel can bring extra demand. When the handling cost is relatively low or mediate, the decision maker can decrease the quality to lower the production cost but not lose demand, and when the handling cost is low or mediate, the decision maker can decrease or increase the price to manage the low or high handling cost. However, when the handling cost is relatively high (but not too high), the decision maker needs to improve the quality to appeal to customers and increase the price to manage the high production cost and handling cost.

5.2 A General Model

In this section, we relax some assumptions made in our base model and aim to show that our main results are still valid. We only consider the case in which the product is available for purchase through online or in-store. One can easily follow the same procedure to investigate the case in which the product is available for purchase only through online. First, we relax the assumption of negligible transaction cost when the customers purchase through the Online or the BOPS channel. Therefore, the net utility of a customer purchasing the product from the Store, Online and BOPS channels can be respectively given by

$$U_s = \theta q - p - t - l_s,\ U_o = \theta q - p - s - l_o,\ U_{bops} = \theta q - p - t - l_o, \tag{5}$$

where $l_o > 0$ is the transaction cost for purchase through the Online or the BOPS channel. Clearly, we have $l_s > l_o$. Second, we consider that the retailer would incur an extra cost for selling the product through the Store channel compared with the Online channel or BOPS channel due to the extra storage/insurance for keeping the product in the store, denoted by $c_s > 0$. Finally, recall that the Store channel is always dominated by the BOPS channel, we thus assume that there are some customers preferring the Store channel, regardless of whether the Online or the BOPS channel is available, which implies that the demand for the Store channel is non-zero when the retailer adds the BOPS channel. Denote by α the proportion of Type 1 customers who prefer the Store channel, and thus $1-\alpha$ the proportion of Type 2 customers who prefer the available channel with the highest utility. Consequently, similar to the base model, the Type 1 customers' demands D_{1s}^n and D_{1s} for the Store channel when the BOPS channel is not offered and offered by the retailer are both given by

$$D_{1s}^n = D_{1s} = \begin{cases} \frac{(q-p-l_s)^2}{2q}, & 0 < q-p-l_s \leq 1, \\ \frac{2q-2p-2l_s-1}{2q}, & q-p-l_s > 1. \end{cases}$$

Moreover, when the BOPS channel is not offered by the retailer, the Type 2 customers' demands for the Online and Store channels are respectively given by $D_{2o}^n = \frac{(q-p-s-l_o)(1-s+l_s-l_o)}{q}$ and $D_{2s}^n = \frac{(2q-2p-s-l_o-l_s)(s+l_o-l_s)}{2q}$; and the profit functions of the retailer and the manufacturer are

$$\pi_r^n = \alpha(p-w-c_s)D_{1s}^n + (1-\alpha)\left[(p-w)D_{2o}^n + (p-w-c_s)D_{2s}^n\right],$$

and

$$\pi_m^n = (w-q^2)[\alpha D_{1s}^n + (1-\alpha)(D_{2o}^n + D_{2s}^n)],$$

respectively. Finally, when the BOPS channel is offered by the retailer, the Type 2 customers' demands for the Online and BOPS channels are respectively given by $D_{2o} = \frac{(q-p-s-l_o)(1-s)}{q}$ and $D_{2bops} = \frac{(2q-2p-2l_o-s)s}{2q}$; and the profit functions of the retailer and the manufacturer are

$$\pi_r = \alpha(p-w-c_s)D_{1s} + (1-\alpha)\left[(p-w)D_{2o} + (p-w-h)D_{2bops}\right],$$

and

$$\pi_m = (w-q^2)\left[\alpha D_{1s} + (1-\alpha)(D_{2o} + D_{2bops})\right],$$

respectively. To avoid trivial cases, we assume that $q-p-s-l_o \geq 0$ and $l_s - l_o \leq s \leq 1$. Following the same procedure as in the base model, one can also obtain the optimal solutions of the general model. However, because it is difficult, if not impossible, to analyze the impacts of adding the BOPS channel, we attempt to conduct numerical experiments to compare the quality, prices and profit performance in the absence and presence of the BOPS channel. To be consistent with reality, we let $\alpha \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ such that the proportion of Type 1 customers who prefer the Store channel is not too large. Furthermore, we let $s, l_s, l_o, c_s \in \{0.005, 0.01, 0.015, 0.02, 0.025\}$ and $h \in \{0.0005, 0.001, 0.0015, 0.002, 0.0025\}$ (thus 5^6=15625 scenarios in total).[11] Our numerical

[11] Note we need some assumptions before the numerical study, for example $l_o < l_s$, the effective number of experiments will thus be less than 15625.

results also suggest that when the handling cost is not too high, a win-win outcome can be reached, i.e., both the retailer and the manufacturer' profits increase compared with the case in which the BOPS channel is not offered. And to achieve the win-win outcome, the manufacturer needs to decrease the wholesale price and the quality, while the retailer should reduce the selling price for low handling cost but increase the price for mediate handling cost. Moreover, when the number of Type 1 customers who prefer the Store channel increases, the retailer should reduce the price for higher handling cost to make more Type 2 customers purchase through the BOPS channel and achieve the win-win outcome.

6 Conclusions

Motivated by the "buy-online-and-pickup-in-store" strategy that has become prevalent in the multi-channel supply chain, we develop a Stackelberg game-theoretic model to study the impact of the BOPS channel on quality, prices and profits of a manufacturer and a retailer. We establish that adding the BOPS channel can lead to a win-win outcome, i.e., both the manufacturer and the retailer are better off, for not too high handling cost through decreasing the product quality and wholesale price but increasing the selling price under certain conditions. When the product is available for purchase only (both) through online (online and in-store), we suggest that adding the BOPS channel results in (higher) lower selling price for a relatively low handling cost if the shipping cost increases. Moreover, when the BOPS channel is not available, our result confirms that adding the Store channel is beneficial for both the manufacturer and the retailer, and they should reduce the quality and wholesale price but increase the selling price when the shipping cost is relatively high. When the Store channel is dominated by the available BOPS channel, however, opening the Store channel may not benefit both parties. In addition, we find that adding the BOPS channel would increase (reduce) both consumer surplus and social welfare for a sufficiently low (high) handling cost. When the decisions are made in a centralized manner, we observe that it is profitable for a decision maker to add the BOPS channel via increasing both the price and quality under some simple conditions. Additionally, we extend our base model to a more general one and illustrate that our main results are robust.

Our model has limitations that deserve discussion and future research. First, for ease of analysis, we assume that the manufacturer produces the product with the same quality at the same wholesale price, and the retailer sells the product to customers at the same price through the three discussed channels. In some cases, however, the manufacturer might produce a product with lower quality and the retailer offers a discount to those customers who purchase from the Online channel (Brynjolfsson and Smith 2000). One may extend our model framework to allow for this price and quality dispersion. Second, we consider that the manufacturer distributes the product to the retailer who decides whether to open the BOPS channel. However, the manufacturer might also open a new channel to compete with the retailer. The impact of the BOPS channel offered by the manufacturer deserves further investigation. Finally, some extant literature studies horizontal and/or vertical competition between multiple manufacturers and multiple retailers. An interesting extension is thus to introduce competition among multiple players in a supply chain or even multiple supply chains and examine the impact of the BOPS channel.

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References

Bernstein F, Song JS, Zheng X (2009) Free riding in a multi-channel supply chain. *Naval Research Logistics* 56(8):745–765.

Bhargava HK, Choudhary V (2001) Information goods and vertical differentiation. *Journal of Management Information Systems* 18(2):89–106.

Biswas I, Avittathur B (2019) Channel coordination using options contract under simultaneous price and inventory competition. *Transportation Research Part E: Logistics and Transportation Review* 123:45–60.

Brynjolfsson E, Smith MD (2000) Frictionless commerce? A comparison of internet and conventional retailers. *Management science* 46(4):563–585.

Cai GG, Zhang ZG, Zhang M (2009) Game theoretical perspectives on dual-channel supply chain competition with price discounts and pricing schemes. *International Journal of Production Economics* 117(1):80–96.

Cao J, So KC, Yin S (2016) Impact of an "online-to-store" channel on demand allocation, pricing and profitability. *European Journal of Operational Research* 248(1):234–245.

Cattani K, Gilland W, Heese HS, Swaminathan J (2006) Boiling frogs: Pricing strategies for a manufacturer adding a direct channel that competes with the traditional channel. *Production and Operations Management* 15(1):40–57.

Chen J, Liang L, Yao DQ, Sun S (2017) Price and quality decisions in dual-channel supply chains. *European Journal of Operational Research* 259(3):935–948.

Chen J, Zhang H, Sun Y (2012) Implementing coordination contracts in a manufacturer stackelberg dual-channel supply chain. *Omega* 40(5):571–583.

Chen X, Wang X (2015) Free or bundled: Channel selection decisions under different power structures. *Omega* 53:11–20.

Chiang WyK, Chhajed D, Hess JD (2003) Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management science* 49(1):1–20.

Desai PS (2001) Quality segmentation in spatial markets: When does cannibalization affect product line design? *Marketing Science* 20(3):265–283.

Du S, Wang L, Hu L (2019) Omnichannel management with consumer disappointment aversion. *International Journal of Production Economics* 215:84 – 101.

Gao F, Su X (2016) Omnichannel retail operations with buy-online-and-pick-up-in-store. *Management Science* 63(8):2478–2492.

Hendershott T, Zhang J (2006) A model of direct and intermediated sales. *Journal of Economics & Management Strategy* 15(2):279–316.

Jin M, Li G, Cheng T (2018) Buy online and pick up in-store: Design of the service area. *European Journal of Operational Research* 268(2):613–623.

Kumar N, Ruan R (2006) On manufacturers complementing the traditional retail channel with a direct online channel. *Quantitative Marketing and Economics* 4(3):289–323.

MacCarthy BL, Zhang L, Muyldermans L (2019) Best performance frontiers for buy-online-pickup-in-store order fulfilment. *International Journal of Production Economics* 211:251 – 264.

Matsui K (2017) When should a manufacturer set its direct price and wholesale price in dual-channel supply chains? *European Journal of Operational Research* 258(2):501–511.

Modak NM, Kelle P (2019) Managing a dual-channel supply chain under price and delivery-time dependent stochastic demand. *European Journal of Operational Research* 272(1):147 – 161.

Moorthy KS (1988) Product and price competition in a duopoly. *Marketing science* 7(2):141–168.

Niu B, Cui Q, Jie Z (2017) Impact of channel power and fairness concern on supplier's market entry decision. *Journal of the Operational Research Society* 68:1–12.

Niu B, Mu Z, Li B (2019) O2O results in traffic congestion reduction and sustainability improvement: Analysis of "online-to-store" channel and uniform pricing strategy. *Transportation Research Part E: Logistics and Transportation Review* 122:481–505.

Shi H, Liu Y, Petruzzi NC (2013) Consumer heterogeneity, product quality, and distribution channels. *Management Science* 59(5):1162–1176.

Shi X, Dong C, Cheng T (2018) Does the buy-online-and-pick-up-in-store strategy with pre-orders benefit a retailer with the consideration of returns? *International Journal of Production Economics* 206:134–145.

Tsay AA, Agrawal N (2004a) Channel conflict and coordination in the e-commerce age. *Production and operations management* 13(1):93–110.

Tsay AA, Agrawal N (2004b) Modeling conflict and coordination in multi-channel distribution systems: A review. *Handbook of quantitative supply chain analysis*, 557–606 (Springer).

Villas-Boas JM (1998) Product line design for a distribution channel. *Marketing Science* 17(2):156–169.

Wang S, Hu Q, Liu W (2017) Price and quality-based competition and channel structure with consumer loyalty. *European Journal of Operational Research* 262(2):563–574.

Xu X (2009) Optimal price and product quality decisions in a distribution channel. *Management Science* 55(8):1347–1352.

Zhang J, Cao Q, He X (2019a) Contract and product quality in platform selling. *European Journal of Operational Research* 272(3):928–944.

Zhang P, He Y, Zhao X (2019b) "Preorder-online, pickup-in-store" strategy for a dual-channel retailer. *Transportation Research Part E: Logistics and Transportation Review* 122:27–47.

Zhang Q, Tang W, Zaccour G, Zhang J (2019c) Should a manufacturer give up pricing power in a vertical information-sharing channel? *European Journal of Operational Research* 276(3):910 – 928.

Zhou J, Zhao R, Wang W (2019) Pricing decision of a manufacturer in a dual-channel supply chain with asymmetric information. *European Journal of Operational Research* 278(3):809 – 820.

Appendix

Proof of Lemma 1: The retailer solves the following optimization problem: $\max_p \pi_r^{on}(p) = (p-w)\frac{q-p-s}{q}$, which is clearly a concave maximization. The first-order optimality condition is

$$\frac{\partial \pi_r^{on}}{\partial p} = \frac{q-2p+w-s}{q} = 0.$$

Solving this equation yields a unique solution $p^* = \frac{q+w-s}{2}$. We next derive the manufacturer's optimal wholesale price. Given any quality q, the manufacturer solves the following optimization problem:

$$\max_w \pi_m^{on}(w) = (w-q^2)D_o^{on}(p^*,q) = \frac{(w-q^2)(q-w-s)}{2q},$$

which is also a concave maximization. The first-order optimality condition is

$$\frac{\partial \pi_m^{on}}{\partial w} = \frac{q^2+q-2w-s}{2q} = 0.$$

Solving this equation yields a unique solution $w^* = \frac{q^2+q-s}{2}$. We now derive the manufacturer's optimal quality. The manufacturer solves the following optimization problem:

$$\max_q \pi_m^{on}(q) = (w^*-q^2)D_o^{on}(p^*,q) = \frac{(q^2-q+s)^2}{8q}.$$

The first-order optimality condition is

$$\frac{\partial \pi_m^{on}}{\partial q} = \frac{(q^2-q+s)(3q^2-q-s)}{8q^2} = 0.$$

Solving the above equation, we obtain $q = \frac{1+\sqrt{1-4s}}{2}$ or $\frac{1-\sqrt{1-4s}}{2}$ or $\frac{1+\sqrt{1+12s}}{6}$ or $\frac{1-\sqrt{1+12s}}{6}$. First, $q = \frac{1-\sqrt{1+12s}}{6} < 0$ is not the optimal solution, and moreover $q = \frac{1+\sqrt{1-4s}}{2}$ or $\frac{1-\sqrt{1-4s}}{6}$ makes the manufacturer's profit zero, which is also not the optimal solution. We next show that $q = \frac{1+\sqrt{1+12s}}{6}$ is the unique optimal quality. It suffices to show that the manufacturer's profit function is concave in the solution $q = \frac{1+\sqrt{1+12s}}{6}$, i.e., $\frac{\partial^2 \pi_m^{on}}{\partial q^2}|_{q=\frac{1+\sqrt{1+12s}}{6}} \leq 0$. In fact, recall that we assume $q - p \geq s$ to guarantee non-negative demand for the Online channel, we thus have $s \leq \frac{1}{4}$. This implies that $\frac{\partial^2 \pi_m^{on}}{\partial q^2}|_{q=\frac{1+\sqrt{1+12s}}{6}} = -\frac{1+6s(1-12s)+\sqrt{1+12s}}{(1+\sqrt{1+12s})^3} \leq 0$. Consequently, we have $q^{on*} = \frac{1+\sqrt{1+12s}}{6}$, $p^{on*} = \frac{5-24s+5\sqrt{1+12s}}{36}$, $w^{on*} = \frac{1-3s+\sqrt{1+12s}}{9}$, $\pi_r^{on*} = \frac{(1-12s+\sqrt{1+12s})^2}{216(1+\sqrt{1+12s})}$ and $\pi_m^{on*} = \frac{(1-12s+\sqrt{1+12s})^2}{108(1+\sqrt{1+12s})}$. □

Proof of Proposition 1: By Lemma 1, the optimal profits of the retailer and the manufacturer are given by $\pi_r^{on*} = \frac{(1-12s+\sqrt{1+12s})^2}{216(1+\sqrt{1+12s})}$ and $\pi_m^{on*} = \frac{(1-12s+\sqrt{1+12s})^2}{108(1+\sqrt{1+12s})}$, respectively. Therefore,

$$\begin{cases} \frac{\partial w^{on*}}{\partial s} = \frac{2-\sqrt{1+12s}}{3\sqrt{1+12s}}, \frac{\partial q^{on*}}{\partial s} = \frac{1}{\sqrt{1+12s}}, \frac{\partial p^{on*}}{\partial s} = \frac{5-4\sqrt{1+12s}}{6\sqrt{1+12s}}, \frac{\partial D_o^{on*}}{\partial s} = -\frac{2(1+6s+\sqrt{1+12s})}{\sqrt{1+12s}(1+\sqrt{1+12s})^2}, \\ \frac{\partial \pi_r^{on*}}{\partial s} = -\frac{(1-12s+\sqrt{1+12s})(1+12s+\sqrt{1+12s})}{12\sqrt{1+12s}(1+\sqrt{1+12s})^2}, \frac{\partial \pi_m^{on*}}{\partial s} = -\frac{(1-12s+\sqrt{1+12s})(1+12s+\sqrt{1+12s})}{6\sqrt{1+12s}(1+\sqrt{1+12s})^2}. \end{cases}$$

According to the assumption $s \leq \frac{1}{4}$, one can easily prove the desired results. □

Proof of Theorem 2: Note first from the proofs of Lemma 1 and Lemma 2, we can know that $0 \leq h \leq \frac{s^2}{2(1-s)}$ and $s \leq \underline{s}$; and $0 \leq h \leq \frac{1-8s-8s^2+\sqrt{1-8s^2}}{8s}$ and $\underline{s} \leq s \leq s$, where $\underline{s} \approx 0.1923$ and $\overline{s} \approx 0.1927$. Let

$$\begin{aligned} \Delta_r = & \frac{1}{12}\left[(-2h+s)\sqrt{1+12s} - 2h - 11s - 1\right]\left[1 + \sqrt{1+6s(2+2h-s)}\right] \\ & + \left[\frac{s^3}{4} + (\frac{7h}{2} - 1)s^2 + (h^2 - \frac{5h}{2} + \frac{25}{24})s - \frac{h}{12} + \frac{1}{12}\right]\sqrt{1+12s} + \frac{s^3}{4} + (\frac{7h}{2} - 1)s^2 + (h^2 - \frac{5h}{2} + \frac{1}{24})s - \frac{h}{12}, \end{aligned}$$

then we have

$$\pi_r^{o*} - \pi_r^{on*} = \frac{2s}{3[1+\sqrt{1+12s}][1+\sqrt{1+6s(2+2h-s)}]}\Delta_r.$$

Plotting the function Δ_r over the regions $h \in [0, \frac{s^2}{2(1-s)}]$ and $h \in [0, \frac{1-8s-8s^2+\sqrt{1-8s^2}}{8s}]$ for any given $s \in (0, \underline{s}]$ and $s \in [\underline{s}, \overline{s}]$, respectively, one can easily know that Δ_r decreases in h, and there exists $\overline{h}$ such that $\pi_r^{o*} - \pi_r^{on*} \geq 0$ if and only if $h \leq \overline{h}$. Moreover, $\overline{h}$ first increases and then decreases in s. Since

$$\pi_m^{o*} - \pi_m^{on*} = \frac{4s}{3[1+\sqrt{1+12s}][1+\sqrt{1+6s(2+2h-s)}]}\Delta_m,$$

and $\Delta_m - \Delta_r = \frac{9}{2}hs(1-s)(1+\sqrt{1+12s}) \geq 0$, where

$$\begin{aligned} \Delta_m = & \frac{1}{12}\left[(-2h+s)\sqrt{1+12s} - 2h - 11s - 1\right]\left[1 + \sqrt{1+6s(2+2h-s)}\right] \\ & + \left[\frac{s^3}{4} - (h+1)s^2 + (h^2 + 2h + \frac{25}{24})s - \frac{h}{12} + \frac{1}{12}\right]\sqrt{1+12s} + (h - \frac{s}{2})[-\frac{1}{12} - \frac{s^2}{2} + (h+2)s], \end{aligned}$$

we have $\pi_m^{o*} - \pi_m^{on*} \geq 0$ for $h \leq \overline{h}$ due to $\pi_r^{o*} - \pi_r^{on*} \geq 0$. $\square$

Proof of Corollary 1: From the proofs of Theorems 1 and 2, it suffices to show that $\overline{h}_p \leq \overline{h} \leq \overline{h}_m = \frac{s}{2}$. Since $\overline{h}$ is the unique positive root of the function Δ_r, and moreover $0 \leq h \leq \frac{s^2}{2(1-s)}$ and $s \leq \underline{s}$; and $0 \leq h \leq \frac{1-8s-8s^2+\sqrt{1-8s^2}}{8s}$ and $\underline{s} \leq s \leq s$, where $\underline{s} \approx 0.1923$ and $\overline{s} \approx 0.1927$, combining with these conditions and plotting the function $\frac{s}{2} - \overline{h}$, we can know that $\frac{s}{2} - \overline{h} \geq 0$. Similarly, $\overline{h}_p \leq \overline{h}$ can be proved. $\square$

Proof of Corollary 2: From the proofs of Lemma 1 and Lemma 2, and Equations (2) and (3), we can obtain the expressions of consumer surplus and social welfare at optimal (note that $s \leq \frac{1}{4}$ and $2 + 2h + s \geq 8s(1 + h + s)^2$). After tedious calculation (for brevity, we omit these lengthy expressions and calculations, which are available from the authors upon request), we can show that there exists a threshold $\overline{h}_{cs}$ such that $CS^{o*} \geq CS^{on*}$ and $LW^{o*} \geq LW^{on*}$ if and only if $h \leq \overline{h}_{cs}$. $\square$

Proof of Lemma 3: The retailer solves the following optimization problem: $\max_p \pi_r^n(p) = (p - w)\left[\frac{(q-p-s)(1-s+l_s)}{q} + \frac{(2q-2p-s-l_s)(s-l_s)}{2q}\right]$, which is clearly a concave maximization. The first-order optimality condition is

$$\frac{\partial \pi_r^n}{\partial p} = \frac{2(q - 2p + w) + \phi}{2q} = 0.$$

Solving this equation yields a unique solution $p^* = \frac{2(q+w)+\phi}{4}$. We next derive the manufacturer's optimal wholesale price. Given any quality q, the manufacturer solves the following optimization problem:

$$\max_w \pi_m^n(w) = (w - q^2)[D_o^n(p^*, q) + D_s^n(p^*, q)] = \frac{(w - q^2)[2(q - w) + \phi]}{4q},$$

which is also a concave maximization. The first-order optimality condition is

$$\frac{\partial \pi_m^n}{\partial w} = \frac{2(q^2 + q - 2w) + \phi}{4q} = 0.$$

Solving this equation yields a unique solution $w^* = \frac{2(q^2+q)+\phi}{4}$. We now derive the manufacturer's optimal quality. The manufacturer solves the following optimization problem:

$$\max_q \pi_m^n(q) = (w^* - q^2)[D_o^n(p^*, q) + D_s^n(p^*, q)] = \frac{[2(q^2 - q) - \phi]^2}{32q}.$$

The first-order optimality condition is

$$\frac{\partial \pi_m^n}{\partial q} = \frac{(2q^2 - 2q - \phi)(6q^2 - 2q + \phi)}{32q^2} = 0.$$

Solving the above equation, we obtain $q = \frac{1+\sqrt{1+2\phi}}{2}$ or $\frac{1-\sqrt{1+2\phi}}{2}$ or $\frac{1-\sqrt{1-6\phi}}{6}$ or $\frac{1+\sqrt{1-6\phi}}{6}$. First, $q = \frac{1-\sqrt{1-6\phi}}{6} < 0$ is not the optimal solution, and moreover $q = \frac{1+\sqrt{1+2\phi}}{2}$ or $\frac{1-\sqrt{1+2\phi}}{2}$ makes the manufacturer's profit zero, which is also not the optimal solution. We next show that $q = \frac{1+\sqrt{1-6\phi}}{6}$ is the unique optimal quality. It suffices to prove that the manufacturer's profit function is concave in the solution $q = \frac{1+\sqrt{1-6\phi}}{6}$, i.e., $\frac{\partial^2 \pi_m^n}{\partial q^2}|_{q=\frac{1+\sqrt{1-6\phi}}{6}} = \frac{(1+3\phi)(-1+6\phi)-\sqrt{1-6\phi}}{(1+\sqrt{1-6\phi})^3} \leq 0$. To avoid trivial cases, we make the following assumption: $-\frac{1}{2} \leq \phi \leq 0$. Under this assumption, one can easily verify that the manufacturer's profit function is concave in the solution $q = \frac{1+\sqrt{1-6\phi}}{6}$. Moreover, recall that we assume $q - p - s = \frac{1+\sqrt{1-6\phi}-12(\phi+3s)}{36} \geq 0$ to guarantee non-negative demand for the Online channel. Consequently, the optimal solutions are as follows: $q^{n*} = \frac{1+\sqrt{1-6\phi}}{6}$, $p^{n*} = \frac{5+12\phi+5\sqrt{1-6\phi}}{36}$, $w^{n*} = \frac{2+3\phi+2\sqrt{1-6\phi}}{18}$, $\pi_m^{n*} = \frac{(1+6\phi+\sqrt{1-6\phi})^2}{108(1+\sqrt{1-6\phi})}$ and $\pi_r^{n*} = \frac{(1+6\phi+\sqrt{1-6\phi})^2}{216(1+\sqrt{1-6\phi})}$. For convenience, we note that combined with the above two assumptions, one can obtain that $s \leq \widetilde{s} \approx 0.1927$, where $\widetilde{s}$ is the solution of $s - \frac{\sqrt{1-16s+\sqrt{1+32s}}}{4} = 0$; and $\widetilde{s} < s \leq 0.25$ and $l_s \geq s - \frac{\sqrt{1-16s+\sqrt{1+32s}}}{4} > 0$. In fact, from $\frac{1+\sqrt{1-6\phi}-12(\phi+3s)}{36} \geq 0$, we have $s_2 \leq \phi \leq 0$ for $s \leq \frac{1}{18}$ and $s_2 \leq \phi \leq s_1$ for $s > \frac{1}{18}$, where $s_1 = -3s + \frac{1+\sqrt{1+32s}}{36}$ and $s_2 = -3s + \frac{1-\sqrt{1+32s}}{36}$. Moreover, $s_1 \geq -\frac{1}{2}$ for $s \leq \frac{1}{4}$ and $s_2 \geq -\frac{1}{2}$ for $s \leq \frac{5}{36}$. This implies that $s_2 \leq \phi \leq 0$ for $s \leq \frac{1}{18}$, $s_2 \leq \phi \leq s_1 < 0$ for $\frac{1}{18} < s \leq \frac{5}{36}$, and $-\frac{1}{2} \leq \phi \leq s_1 < 0$ for $\frac{5}{36} < s \leq \frac{1}{4}$. Since $s_1 - \phi = -s - (s-l_s)^2 + \frac{1+\sqrt{1+32s}}{16}$, one can easily know that $s_1 - \phi$ is increasing in l_s and thus $s_1 - \phi \geq 0$ is equivalent to $l_s \geq s - \frac{\sqrt{1-16s+\sqrt{1+32s}}}{4}$. Combined with $\phi - (-\frac{1}{2}) = (s-l_s)^2 + \frac{1}{2} - 2s > 0$ for $s \leq \frac{1}{4}$, and $\phi - s_2 = (s-l_s)^2 + s + \frac{\sqrt{1+32s}-1}{16} > 0$, and $l_s > 0$ and $s > l_s$, the above statement follows. $\square$

Proof of Proposition 3: We first analyze the impact of s. According to the proof of Lemma 3, one can easily have $\frac{\partial q^{n*}}{\partial l_s} = -\frac{s-1-l_s}{\sqrt{1-6\phi}} \geq 0$, $\frac{\partial w^{n*}}{\partial l_s} = \frac{-2+\sqrt{1-6\phi}}{3\sqrt{1-6\phi}}(s-1-l_s) \geq 0$, $\frac{\partial \pi_m^{n*}}{\partial l_s} = \frac{1-3\phi-18\phi^2+\sqrt{1-6\phi}}{3\sqrt{1-6\phi}(1+\sqrt{1-6\phi})^2}(s-1-l_s) \leq 0$, $\frac{\partial \pi_r^{n*}}{\partial l_s} = \frac{1-3\phi-18\phi^2+\sqrt{1-6\phi}}{6\sqrt{1-6\phi}(1+\sqrt{1-6\phi})^2}(s-1-l_s) \leq 0$, and $\frac{\partial (D_o^{n*}+D_s^{n*})}{\partial l_s} = \frac{1-3\phi+\sqrt{1-6\phi}}{\sqrt{1-6\phi}(1+\sqrt{1-\phi})^2} 2(s-1-l_s) \leq 0$; Since $\frac{\partial p^{n*}}{\partial l_s} = \frac{-5+4\sqrt{1-6\phi}}{6\sqrt{1-6\phi}}(s-1-l_s)$, we obtain that $\frac{\partial p^{n*}}{\partial s} \geq 0$ for $-\frac{3}{32} \leq \phi \leq 0$ and $\frac{\partial p^{n*}}{\partial s} < 0$ for $-\frac{1}{2} \leq \phi < -\frac{3}{32}$. This implies that $\frac{\partial p^{n*}}{\partial s} \geq 0$ if and only if $s \leq \widetilde{s}_0 = l_s + \frac{8-\sqrt{58+128l_s}}{8}$. Consequently, when $l_s > \frac{3}{64}$, we always have $\frac{\partial p^{n*}}{\partial s} \leq 0$; but when $l_s \leq \frac{3}{64}$, we have $\frac{\partial p^{n*}}{\partial s} \geq 0$ for $s \leq \widetilde{s}_0$ and $\frac{\partial p^{n*}}{\partial s} \leq 0$ for $s \geq \widetilde{s}_0 = l_s + \frac{8-\sqrt{58+128l_s}}{8}$, where $\widetilde{s}_0$ decreases in l_s.

We now investigate the impact of l_s. First, one can easily have $\frac{\partial q^{n*}}{\partial s} = -\frac{l_s-s}{\sqrt{1-6\phi}} \geq 0$, $\frac{\partial w^{n*}}{\partial s} = \frac{-2+\sqrt{1-6\phi}}{3\sqrt{1-6\phi}}(l_s-s) \geq 0$, $\frac{\partial \pi_m^{n*}}{\partial s} = \frac{1-3\phi-18\phi^2+\sqrt{1-6\phi}}{3\sqrt{1-6\phi}(1+\sqrt{1-6\phi})^2}(l_s-s) \leq 0$, $\frac{\partial \pi_r^{n*}}{\partial s} = \frac{1-3\phi-18\phi^2+\sqrt{1-6\phi}}{6\sqrt{1-6\phi}(1+\sqrt{1-6\phi})^2}(l_s-s) \leq 0$, and $\frac{\partial (D_o^{n*}+D_s^{n*})}{\partial s} = \frac{1-3\phi+\sqrt{1-6\phi}}{\sqrt{1-6\phi}(1+\sqrt{1-\phi})^2} 2(l_s-s) \leq 0$; Since $\frac{\partial p^{n*}}{\partial s} = \frac{-5+4\sqrt{1-6\phi}}{6\sqrt{1-6\phi}}(l_s-s)$, we obtain that $\frac{\partial p^{n*}}{\partial l_s} \geq 0$ for $-\frac{3}{32} \leq \phi \leq 0$ and $\frac{\partial p^{n*}}{\partial l_s} < 0$ for $-\frac{1}{2} \leq \phi < -\frac{3}{32}$. This implies that $\frac{\partial p^{n*}}{\partial l_s} \geq 0$ if and only if

$l_s \le \widetilde{l}_0 = s - \frac{\sqrt{128s-6}}{8}$. Consequently, when $s \le \frac{3}{64}$, we always have $\frac{\partial p^{n*}}{\partial l_s} \ge 0$; when $\frac{3}{64} < s \le 1 - \frac{\sqrt{58}}{8}$, we have $\frac{\partial p^{n*}}{\partial l_s} \ge 0$ for $l_s \le \widetilde{l}_0$ and $\frac{\partial p^{n*}}{\partial l_s} \le 0$ for $l_s \ge \widetilde{l}_0 = s - \frac{\sqrt{128s-6}}{8}$, where $\widetilde{l}_0$ decreases in s; and when $s > 1 - \frac{\sqrt{58}}{8}$, we have $\frac{\partial p^{n*}}{\partial l_s} \le 0$. □

Proof of Theorem 3: Since

$$\begin{cases} q^* - q^{n*} = \frac{\sqrt{1+6s(2+2h-s)} - \sqrt{1-6[s^2-2(1+l_s)s+l_s^2]}}{6}, \\ w^* - w^{n*} = \frac{\sqrt{1+6s(2+2h-s)} - \sqrt{1-6[s^2-2(1+l_s)s+l_s^2]}}{9} - \frac{l_s^2-2sl_s+2hs}{6}, \\ p^* - p^{n*} = \frac{5[\sqrt{1+6s(2+2h-s)} - \sqrt{1-6[s^2-2(1+l_s)s+l_s^2]}]}{36} + \frac{hs+2sl_s-l_s^2}{3}, \end{cases}$$

we have q^*-q^{n*} and p^*-p^{n*} are increasing in h, and w^*-w^{n*} also increases in h because $\frac{\partial(w^*-w^{n*})}{\partial h} = \frac{s[2-\sqrt{1+6s(2+2h-s)}]}{3\sqrt{1+6s(2+2h-s)}} \ge 0$. Therefore, when $\sqrt{1+6s(2+2h-s)} - \sqrt{1-6[s^2-2(1+l_s)s+l_s^2]} \le 0$, i.e., $h \le \widetilde{h}_m := \frac{l_s(2s-l_s)}{2s} < \frac{s}{2}$, we have $q^* \le q^{n*}$ and $w^* \le w^{n*}$. Clearly, there exists a threshold $\widetilde{h}_p < \widetilde{h}_m$ such that $p^* - p^{n*} \le 0$ if and only if $h \le \widetilde{h}_p$ because $q^* - q^{n*} \ge 0$ can lead to $p^* - p^{n*} \ge 0$. From the proofs of Lemma 3 and Lemma 4, we can know that $0 \le h \le \frac{s^2}{2(1-s)}$ and $s \le \underline{s}$ and $l_s < s$; and $0 \le h \le \frac{1-8s-8s^2+\sqrt{1-8s^2}}{8s}$ and $\underline{s} \le s \le s$ and $l_s < s$, where $\underline{s} \approx 0.1923$ and $\overline{s} \approx 0.1927$. Consequently, plotting the function $\widetilde{h}_p$, we can know that $\widetilde{h}_p$ increases in l_s, but decreases in s. □

Proof of Theorem 4: Note first from the proofs of Lemma 3 and Lemma 4, we can know that $0 \le h \le \frac{s^2}{2(1-s)}$ and $s \le \underline{s}$ and $l_s < s$; and $0 \le h \le \frac{1-8s-8s^2+\sqrt{1-8s^2}}{8s}$ and $\underline{s} \le s \le \overline{s}$ and $l_s < s$, where $\underline{s} \approx 0.1923$ and $\overline{s} \approx 0.1927$. Plotting the function $\pi_r^* - \pi_r^{n*}$ over the regions $h \in [0, \frac{s^2}{2(1-s)}]$ and $h \in [0, \frac{1-8s-8s^2+\sqrt{1-8s^2}}{8s}]$ for any given $s \in (0, \underline{s}]$ and $s \in [\underline{s}, \overline{s}]$ and $l_s \in (0, s)$, respectively, one can easily know that $\pi_r^* - \pi_r^{n*}$ decreases in h, and there exists $\widetilde{h}$ such that $\pi_r^* - \pi_r^{n*} \ge 0$ if and only if $h \le \widetilde{h}$. Moreover, $\widetilde{h}$ increases in l_s but decreases in s. Since

$$\frac{1}{2}(\pi_m^* - \pi_m^{n*}) - (\pi_r^{o*} - \pi_r^{on*}) = \frac{3hs^2(1-s)}{1+\sqrt{1+6s(2+2h-s)}} > 0,$$

we have $\pi_m^* - \pi_m^{n*} \ge 0$ for $h \le \widetilde{h}$ due to $\pi_r^* - \pi_r^{n*} \ge 0$. □

Proof of Corollary 3: From the proofs of Theorems 3 and 4, it suffices to show that $\widetilde{h}_p \le \widetilde{h} \le \widetilde{h}_m = \frac{l_s(2s-l_s)}{2s}$. Since $0 \le h \le \frac{s^2}{2(1-s)}$ and $s \le \underline{s}$ and $l_s < s$; and $0 \le h \le \frac{1-8s-8s^2+\sqrt{1-8s^2}}{8s}$ and $\underline{s} \le s \le s$ and $l_s < s$, where $\underline{s} \approx 0.1923$ and $\overline{s} \approx 0.1927$, plotting the function $\frac{l_s(2s-l_s)}{2s} - \widetilde{h}$, we can know that $\frac{l_s(2s-l_s)}{2s} - \widetilde{h} \ge 0$. Similarly, $\widetilde{h}_p \le \widetilde{h}$ can be proved. □

Proof of Corollary 4: Note first from the proofs of Lemma 3 and Lemma 4, we can know that $0 \le h \le \frac{s^2}{2(1-s)}$ and $s \le \underline{s}$ and $l_s < s$; and $0 \le h \le \frac{1-8s-8s^2+\sqrt{1-8s^2}}{8s}$ and $\underline{s} \le s \le \overline{s}$ and $l_s < s$, where

$\underline{s} \approx 0.1923$ and $\overline{s} \approx 0.1927$. According to Equations (2) and (4), we can obtain the expressions of consumer surplus and social welfare at optimal. Plotting the function $CS^* - CS^{n*}$ and $SW^* - SW^{n*}$ over the regions $h \in [0, \frac{s^2}{2(1-s)}]$ and $h \in [0, \frac{1-8s-8s^2+\sqrt{1-8s^2}}{8s}]$ for any given $s \in (0, \underline{s}]$ and $s \in [\underline{s}, \overline{s}]$ and $l_s \in (0, s)$, respectively, one can know that both $CS^* - CS^{n*}$ and $SW^* - SW^{n*}$ decrease in h, and there exists $\overline{h}_{cs1}$ ($\overline{h}_{cs2}$) such that $CS^* - CS^{n*} \geq 0$ and $SW^* - SW^{n*} \geq 0$ ($CS^* - CS^{n*} < 0$ and $SW^* - SW^{n*} < 0$) for $h \leq \overline{h}_{cs1}$ ($h > \overline{h}_{cs2}$). $\square$