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Phase space elliptic density feature for epileptic EEG signals classification using metaheuristic optimization method

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A B S T R A C T

The electroencephalography (EEG), which is a method for monitoring the brain signals, is a common method used to diagnose the epileptic seizures. In this study, some features are presented for the classification of the brain signals. These features are based on the texture and structure of the brain signals in the phase space representation (PSR). Due to the resonance property, the data are elliptical in the phase space. Therefore, the mentioned features are based on the calculations of the data density in the ellipses.

In the first method of feature extraction, the radius values of the ellipses are assumed based on the normal distribution feature. In the two other methods of the presented features, the radius values of the assumed ellipses are calculated by the optimizer. These methods of feature extraction are based on the incremental ellipses and intersecting ellipses, respectively. The density of the data in the assumed ellipses is given to the k-nearest neighbor as a feature to classify the epileptic seizure and seizure-free EEG signals.

The intended method was implemented and investigated on two databases of the Bonn university of Germany and the neurology and sleep center of New Delhi. The results indicate that the proposed features are strong tools for separation and diagnosis of this type of signal and have higher accuracy compared to the other classic and updated methods. The extraction speed of the presented features was higher in the test phase compared to the other methods.

1. Introduction

The diagnosis of epilepsy disorder is one of the most used actions in the medical and neurology field [1]. Generally, the epilepsy is described as a disorder in the central nervous system, caused by a sudden and severe discharge of the brain neurons [2,3]. Around one percent of the people in the world have epilepsy [4]. Researchers are looking for a reliable solution for the prediction and diagnosis of epileptic seizures by examining the EEGs of patients with epilepsy [5,6].

There are waves with very high frequencies and heights higher than the mV range in the epileptic seizure step [7]. These fast and high waves, which are the characteristics of the epileptic seizure, are called a spike [8–10]. These spikes are the main indices for the presence of the epileptic seizures. There are methods based on the feature extraction techniques to diagnose these spikes from the brain electroencephalograph automatically. These techniques are divided into four groups, including time-domain, frequency-domain, time–frequency-domain, and the nonlinear signal analysis method [4].

Assuming the EEG signals being stationary, the time-domain and frequency-domain features are used to diagnose and classify the epileptic seizures [4,11,12]. Methods that are based on the Linear Prediction (LP) and Fractional Linear Prediction (FLP) are developed for the diagnosis of the epileptic seizures in the EEG signals [12,13]. Principal Component Analysis (PCA) using the enhanced cosine radial basis function neural network method is developed in [11] for the diagnosis of the seizures.

The time–frequency methods based on non-stationary EEG signals are developed in [14–17]. In [18], the simulation of time–frequency analysis is presented for classification of the sections of the epileptic seizure brain signals, and short-term flourier transform and several time–frequency distributions are used to calculate the Power Spectrum Density (PSD) of each section. In [19–25], the analysis and classification methods for EEG epileptic signals are considered based on the wavelet transform and multi-wavelet transform. Also, some features, which use the wavelet coefficient for classification of healthy and epileptic persons, are extracted based on the Euclidian distance in the phase space reconstruction [26]. The studies have shown that the nonlinear parameters, like the approximate entropy (ApEn), are useful for diagnosis and identification of the epileptic seizure brain signals [27]. The reason is the approximate entropy values decrease

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sharply at the time of an epileptic seizure. This value has a significant relationship with the simultaneous discharge of large groups of neurons [28]. The Lyapunov exponent can present considerable details about the changes in the EEG activity and simplify the early diagnosis of epilepsy [21,29]. The fractal dimension parameter is effectively used to track the chaotic nature of the EEG signals for the diagnosis of epileptic seizures [30]. Features obtained by the complexity analysis and spectral analysis of the EEG signals are effectively used to diagnose epilepsy [31].

Methods that are based on empirical mode decomposition (EMD) are developed for the analysis of the nonlinear and non-stationary EEG signals to diagnose and classify the epileptic seizures in the EEG signals [32]. In [3], the empirical mode decomposition is used to obtain the intrinsic mode functions (IMFs). Then, these IMFs are transferred to the phase space. Two horizontal and vertical ranges indicating the extent of data in each IMF is selected as a feature using the 95% confidence ellipse method. These features are applied to the LSSVM classifier. Therefore, the matrix of features gives only information about the total extent of the signal in the phase space to the classifier. It does not transfer any information about the area density and the manner of expansion. Also, the usage of the EMD method for decomposing the signal has processing and timing costs. This cost is applied while there is no significant difference between implementing this feature extraction method in the IMFs and the main signal.

In this work, three types of features are considered for the classification of the brain signals and diagnosing epilepsy. These features are calculated by the main signal map in the phase space representation. Usually, phase space representation techniques require computation of the time lag and embedding dimension parameters for every signal [33,34]. However, a constant lag is considered in this work. The used phased space is a 2D space with V_k and V_{k+1} axes. In this space, signals are elliptical. Now, with the assumption of the ellipses with different radiuses, the number of points in each one of ellipses (the internal density of the ellipses) is stated as the new feature. The difference between these three types is in the manner of calculating the radiuses of the ellipses. Each method is more mature and better than the previous methods.

In the first method, the characteristics of the Gaussian distribution of data is used. In the next two feature extraction methods, an optimizer is used to calculate the optimal radiuses. These two methods obtain the overlapping and intersecting ellipses, respectively.

Eventually, the rate and accuracy of the proposed method are compared with the other conventional methods of epilepsy diagnosis.

2. Proposed method

2.1. Dataset

In this study, two independent EEG datasets are used for the evaluation of the efficiency of the proposed methods. Particularly, the first dataset was registered in the Bonn University [35], and the other one, which is called the second dataset, was obtained from the Neurology and Sleep Center of the Hauz Khas, New Delhi [36].

One of the most well-known databases, which is being used in most of the studies for the automatic diagnosis of epilepsy in the EEG signals, is the database that is used in [35]. This database consists of five groups, including F, N, O, Z, and S. Each group composed of a hundred single-channel samples of EEG signals with a length of 6.23 s a sampling rate of 61.173 Hz that are separated from the continuous electroencephalography. The groups Z and O consist of the samples of healthy persons in a relaxed state with eyes open (Z) and closed eyes (O). The groups N and F are samples of the patients in the seizure-free intervals. The group S consists of the electroencephalography signals of the patients during seizures, which are recorded using the EEG device. All of the used signals are recorded using a 128-channel system. A sample of each group is shown in Fig. 1.

The artifacts of this dataset are eliminated by visual examination using a band-pass filter with the frequencies from 40 to 53 Hz. Four categories, including S–Z, S–N, S–F, and S–ZNOF, are considered to diagnose the epileptic and healthy signals according to the mentioned explanations.

The second dataset that belongs to the Neurology and Sleep Center of New Delhi consists of EEG signals of ten patients with epilepsy [37]. These signals were recorded by a Grass Telefactor Comet AS40 amplification system and 16 gold-coated electrodes. The position of the electrodes was in accordance with the 10–20 standard. The sampling rate was 200 Hz. The signals were filtered using the band filter with the cut-off frequencies of 5.0 Hz and 70 Hz.

The dataset was divided into three sections, including pre-ictal (subset A), inter-ictal (subset B), and ictal (subset C), and each subset consists of 50 sections. The length of each section is 12.5 s, which is in accordance with 1024 samples. An example of each section is shown in Fig. 2. The categories of A–C, B–C, A–B, and AB–C are used to evaluate the efficiency of the proposed features in the classifier.

2.2. Phase space representation

In comparison with the uneven spaced time series, the usage of linear methods has the following problems. First, it fails to diagnose the dynamic state and its structure, and second, it does not represent much information about the dynamic state. Also, it does not distinguish noise from chaos [38].

The applications of the reconstructed phase space (RPS) and the chaos theory have attracted much attention from scientists of medical sciences, physicists, mathematicians, and engineers [3,22,39–41]. The phase space is reconstructed based on the delay coordinated. Since a nonlinear dynamic system cannot be divided into smaller parts and solve each one of them separately, the whole system should be studied and investigated all together. The reconstructed phase space (RPS) is a very useful tool for the extraction of the nonlinear dynamic state of the signal. Phase space representation of a signal provides an intuitive image of the evolution of the dynamic behavior of the signal during the time.

The electroencephalograms can be written as a vector of time series, $V_K = \{v_1, v_2, \ldots, v_N\}$, where N is total number of data points and v_i is the ith element of the time series [42]. The phase space is reconstructed by its pathway, which can be stated as below:

$$Y_K = (V_K, V_{K+\tau}, \ldots, V_{K+(d-1)\tau}) \tag{1}$$

where, $K = 1, 2, \ldots, N - (d - 1)\tau$, τ is the delay value, and d is the number of selected dimensions [43]. V_{K+i} is a vector with data points of differences between original corresponding data point and its ith next data point and it can be shown as $V_{K+i} = \{v_1 - v_{1+i}, v_2 - v_{2+i}, \ldots, v_{N-i} - v_N\}$. The dynamic behavior of the signal used in the proposed method is considered to be two dimensional, and the phase space is reconstructed based on $\tau = 1$ [42]. In this case, signal converts to two vectors (dimensions), which the second vector is a right-shifted V_K by one value and subtracted from V_K.

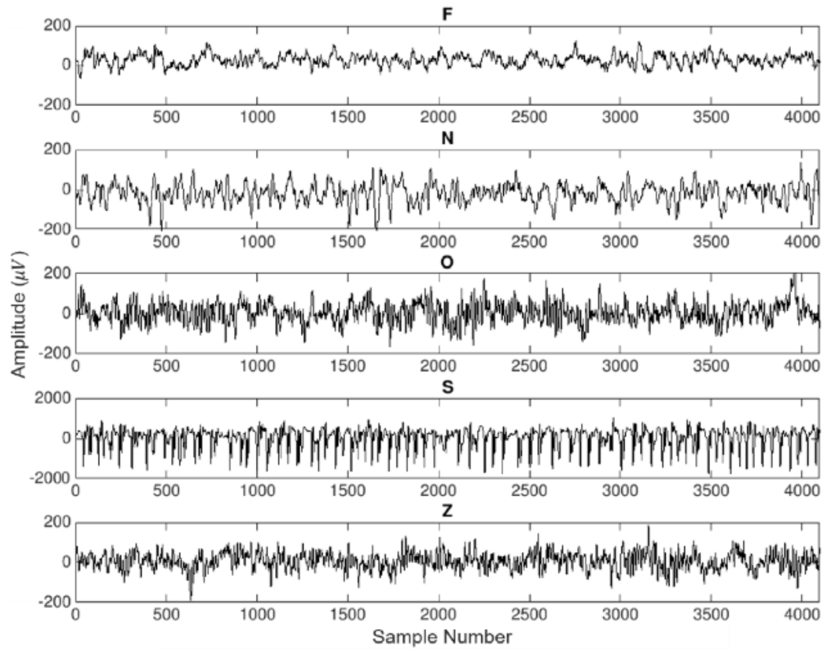

Fig. 1. Sample of S, Z, N, O, F classes in Bonn University dataset.

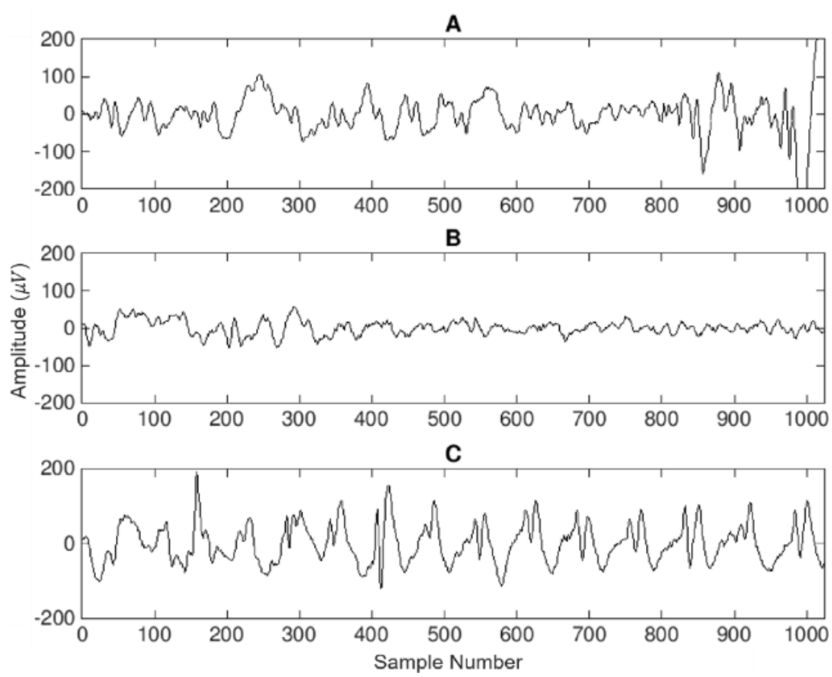

Fig. 2. Sample of A, B, C classes in Hauz Khas dataset.

2.3. Feature extraction and the proposed algorithm

According to previous studies, phase space can present diagnostic features for the classification of the epileptic and healthy signals. It is demonstrated that the sinusoidal signals can generate an elliptical pattern in the phase space [39]. Therefore, due to the oscillating nature of the signals, it is expected for the EEG signals to have an elliptical pattern in the phase space [3]. A sample of phase space representation diagram for the brain signals is shown in Fig. 3.

The principal component analysis is used to calculate the center of this elliptical pattern. This nonlinear transition transfers the center of the signal phase space diagram to point zero, and also makes the angle of the diagram with the horizon line zero. Then, assuming a number of ellipses with different radiuses with the center of the figure, the signal data density in these ellipses can be considered as a feature. In order to calculate the radiuses of the optimal ellipses, the datasets are divided into two sections, including training and testing.

Three innovational methods are considered to apply the ellipses to the training dataset, which is explained in the next section. After calculating the optimal radius of the ellipses, these radiuses are applied to the testing dataset. The number of data in each signal, which is in the assumed ellipse, is given to the classifier as a feature, according to the selected innovational methods.

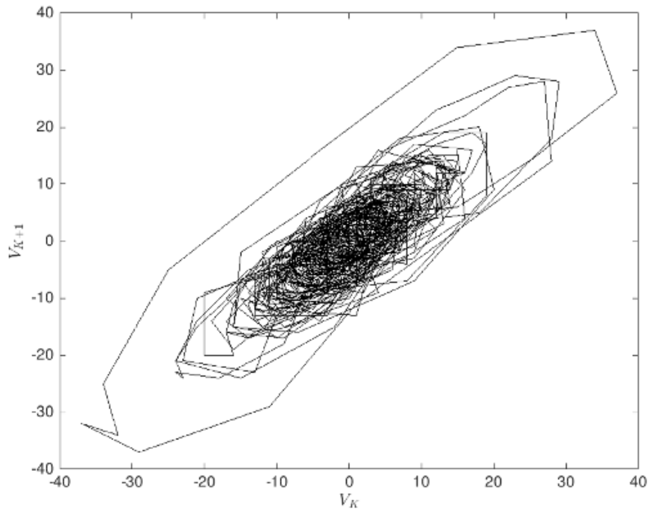

Fig. 3. EEG signal in 2D phase space representation.

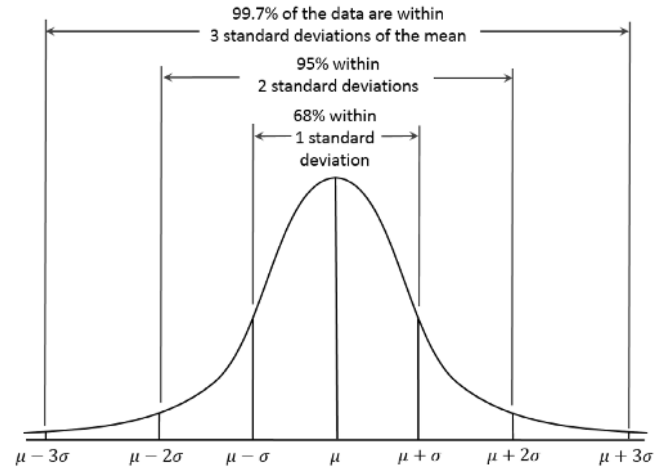

Fig. 4. Distributed of data in Gaussian model.

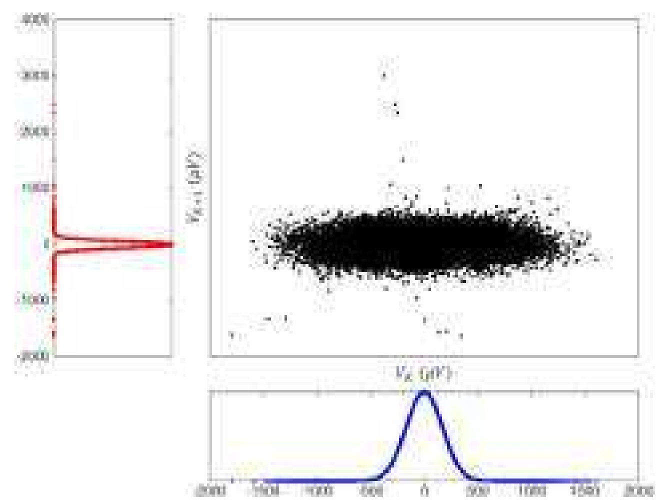

Fig. 5. Distribution of horizontal and vertical axes of signal in phase space representation..

The classifier used in this work is the classic k-nearest neighbor classifier with Euclidian distance and neighbor number (k) of 1, which is one of the most basic types of classifiers. This classifier is used to show the capabilities of the stated feature extraction and its power compared to the other feature extraction methods that use more complex classifiers. In order to obtain an unbiased evaluation of the classification performance, 10-fold cross validation is used to evaluate the precision of the classification.

Algorithm 1. Base EEG epillepic classification

Input: *dataset with labels*

Output: *accuracy*

1: split data into train and test.

2: extract ellipses radiuses from chosen feature extraxtion method based on train data.

3: apply radiuses to test data

4: count number of data is in assumed ellipses as featuresfor all tran and test data

5: classify test data with 1-nearest neighbor classifier

6: compute accuracy of test data

2.3.1. Gaussian elliptic density (Gaussian ED)

Most of the natural phenomena have a Gaussian distribution feature. The importance of this distribution is due to the consistency of many of the calculated values during natural and physical oscillations around a constant value with the values calculated by this distribution. The probability density function of this distribution is as below:

$$n\left(x;\bar{x},\sigma\right)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^{2}} \tag{2}$$

where $\bar{x}$ is the mean data, and σ is the standard deviation of the data [44].

One of the remarkable properties of the Gaussian distribution is the manner of data distribution in it. Fig. 4 shows this distribution well. For a normal distribution, the values with distances less than the standard deviation from the average are about 68.27% of the set. However, this value for two standard deviations from the average is around 95.45%, and for three standard deviations is 99.73%.

In this method, all of the training data signals of each class is transferred to the phase space. Then, the standard deviation of each one of the classes is separately calculated in the horizontal and vertical direction of PSR. The distribution of a signal in dataset is specified in the vertical and horizontal axes of Fig. 5, where the red diagram is the semi Gaussian distribution of the signal in the v_{k+1} axis, and the blue diagram is the semi Gaussian distribution of the signal in the v_k axis.

Eq. (3) is used to calculate the standard deviation.

$$\sigma=\sqrt{\frac{\sum(x-\bar{x})^{2}}{n}} \tag{3}$$

where n is the number of data.

After transferring to the phase space, due to the application of the principal component analysis method, the average value of the data is a small number close to zero. Therefore, $\bar{x}$ can be removed from this equation. The Eq. (3) is applied vertically and horizontally, and the ascending value of the ellipse radiuses is a multiple of the standard deviation. The radiuses of the ellipses can be calculated by Eq. (4), where R_x^i represents the horizontal radius

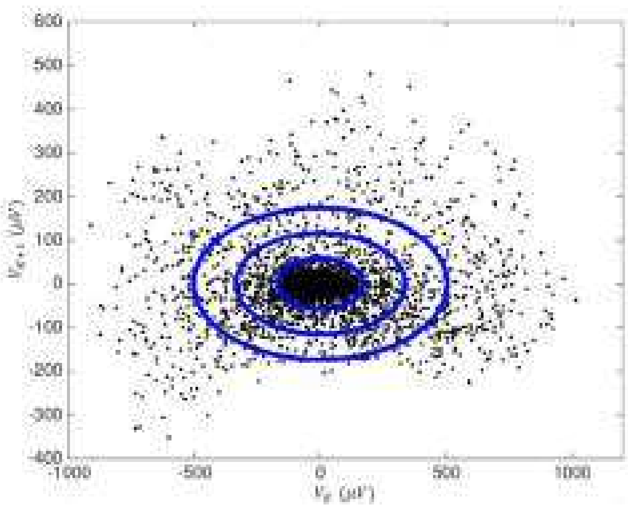

Fig. 6. Ellipses of Gaussian ED on sample data in 2D PSR.

of the ith ellipse, and σ_x and σ_y are the horizontal and vertical standard deviation of the phase diagram, respectively.

$$R_x^i = i \times \sigma_x \quad , \quad R_y^i = i \times \sigma_y \tag{4}$$

According to Figs. 4–6, the number of assumed ellipses is considered to be from 1 to 3, so that 99.73% of the data can be covered. Therefore, i = 1, 2, 3.

Algorithm 2. Gaussian Elliptic Density

Input: *train data,*
n: number of ellipses (1, 2, 3)
Output: *radiuses of ellipses (an array with length of class numbers × n for x axis and y axis)*

```
calculate standard deviation in x-axis and y-axis of each classes
for each classe c do
  for i = 1 to n do
    for each signal s in class c do
      radiuses of ellipse s,i,x = i * standard deviation of signal s by x axis.
      radiuses of ellipse s,i,y = i * standard deviation of signal s by y axis.
    end for
    radiuses of ellipses c,i,1 =mean(radiuses of ellipse :,i,x)
    radiuses of ellipses c,i,2 =mean(radiuses of ellipse :,i,y)
  end for
end for
```

2.3.2. Incremental elliptic density (Incremental ED)

In the method of the previous section, the value of the ellipse radiuses is a multiple of the standard deviation which is a constant value. Now, this raises this idea that what if the distance between the ellipse radiuses is variable? This idea leads to the implementation of a new feature extraction method. Incremental ED is a set of the separate density of each one of the assumed ellipses. According to the previous section, the center of the signal diagram in the phase space is on the zero-point. In this method, the ellipse radiuses are increased growingly. Therefore, the perimeter of the ellipses has no intersections with each other. The radius of the ellipse can be calculated by Eq. (5), where R_x^j and R_y^j are the horizontal and vertical radiuses of the jth ellipse, respectively. Since the searching range is in the positive space, the ellipses would have no intersections but can take different radiuses to increase a better accuracy than the first method.

$$R_x^j = \sum_{i=1}^{j-1} R_x^i \quad , \quad R_y^j = \sum_{i=1}^{j-1} R_y^i \tag{5}$$

The problem here is that how the horizontal and vertical radiuses should be quantified. The answer to this question lies in the purpose of the extraction of this method. The radius quantifications should be such that the fault of the classifier in the validation dataset is decreased and minimized. To this end, the grey wolf optimizer (GWo) is used [45].

GWo has high local optima avoidance which able to provide highly competitive results compared to well-known heuristics such as PSO, GSA, DE, EP, and ES. Finally this optimizer can show high performance on semi-real constrained and real problems [45]. This optimizer iterate 200 times, where coefficient component of 'a', a hyper parameter of GWo algorithm, is linearly decreased from 2 to 0 over the course of iterations. A $2N_E$ array called *particle* is used for coding of the particles (the wolves) that are in the searching space, where N_E is the number of assumed ellipses. 100×*dimension* is the number of search agents, where dimension is the size of particle array. These agents search in space of candidate solutions to find optimum values for every element of particle array in range of signals amplitude. The horizontal and vertical radiuses of the ellipse can be calculated based on Eq. (6).

$$\begin{aligned} R_x^j &= \sum_{i=1}^{j} particle\,(2i-1) \\ R_y^j &= \sum_{i=1}^{j} particle(2i) \end{aligned} \tag{6}$$

where R_x^j and R_y^j represent the horizontal and vertical radiuses of the jth ellipse, respectively.

The input of the fitness function is the particle and the training data of each class. According to the particle array and Eq. (6), the assumed ellipses are drawn on each one of the signal series, and the number of data inside each ellipse is stored as the feature. These features are applied to k-nearest neighbor classifier with 10-fold cross validation on train data and misclassification rate can be considered as the output of the fitness function. The optimizer should work to decrease this rate as particle quantification in next iterations, and at the end of these iterations, the optimal radiuses are obtained (Fig. 7).

Algorithm 3. Fitness function in Incremental Elliptic Density

Input: *train data,*
particle: array with number of wolves length,
n = number of ellipses
Output: *error: 1 – accuracy*

```
feature: matrix of number of data in each ellipse for each signal
for each train signal do
  for i = 1 to n do
        calculate radiuses based on Equation 5
        add row to feature matrix with number of data point in ellipse
  end for
end for
accuracy = knn(feature , label) with 10-fold cross validation
```

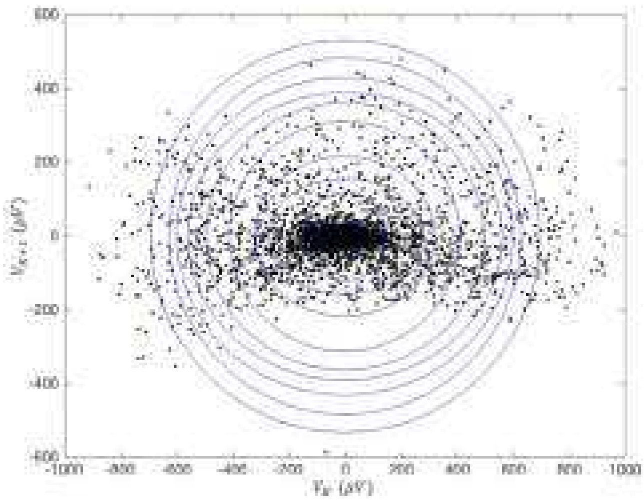

Fig. 7. Ellipses of incremental ED on sample data in 2D PSR.

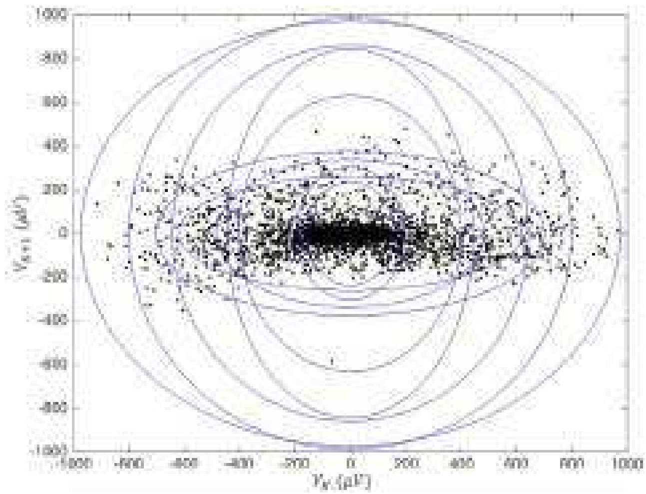

Fig. 8. Ellipses of intersecting ED on sample data in 2D PSR.

2.3.3. *Intersecting elliptic density (Intersecting ED)*

In the previous section, it was mentioned that the ellipses have no intersections. In this feature extraction method, radiuses of the ellipses are selected randomly, and there is the possibility of intersections in assumed ellipses. As in the previous section, an optimizer is used for the calculation of the radiuses to increase the accuracy of the classifier. The particle coding of the optimizer is equal to an array named particle and is twice the size of the ellipses. The radius of the ellipses can be calculated by Eq. (7), where R_x^j and R_y^j are the horizontal and vertical radiuses of the jth ellipse, respectively (Fig. 8).

$$\begin{aligned} R_x^j &= particle(2j-1) \\ R_y^j &= particle(2j) \end{aligned} \tag{7}$$

The fitness function is fed with the inputs of particle and training data of two groups, and its output is misclassification rate for the k-nearest neighbor with 10-fold cross validation. The grey wolf optimizer should find particles to minimize this misclassification in each iteration. Parameter tuning of this optimizer is the same as part 2–3–2.

The fitness function for this method is stated in Algorithm 4.

Algorithm 4. Fitness function in Intersecting Elliptic Density

Input: *train data,*
    particle: array with number of wolves length,
    n = number of ellipses
Output: *error: 1 – accuracy*
feature: matrix of data -counting in each ellipse for each signal
for *each train signal* ***do***
    for *i = 1 to n* ***do***
        calculate radiuses based on Equation 7
        add row to feature matrix with number of data point in ellipse
    end for
end for
accuracy = knn(feature , label) with 10-fold cross validation

3. Results and discussion

In this section, the proposed method of this article is evaluated using three tests. These tests include parameter tuning of the number of ellipses, the results of the different methods stated in this article, and, eventually, a comparison with other methods in terms of speed and accuracy.

3.1. *First test — parameter tuning for the number of ellipses*

In the previous section, the calculation of the optimal value for radiuses in three mentioned feature extraction methods was addressed. However, no explanations were given about the number of formative ellipses. In first method, Gaussian ED, according to the Gaussian characteristics, about 95% of the data is supported in the distance of 2 standard deviations from the average, and 99.7% of data is supported in the distance of 3 standard deviations from the average. Therefore, it is assumed that two or three ellipses are suitable for the classifier. In order to obtain this hyperparameter, a set of data is considered as the validation. Validation is obtained through the training data by the 10-fold cross validation. After calculating the radiuses of the ellipses in the training data, the algorithm is applied to the validation data. The obtained accuracy is given in Table 1. According to the table, number of three ellipses has the highest average classifier accuracy, which is 89.03 percent.

In the following, two ellipses are used for classification with this method. The matrix of features consists of two ellipses with a radius equal to the coefficients of standard deviation in first class training data, and two ellipses with a radius equal to the coefficients of standard deviation in second class training data. Which means that the matrix of features has four columns. In the second and third feature extraction techniques, the number of ellipses is obtained by the trial and error method on the validation data. The number of ellipses is first tested as 2, and depending on the data, the best number of ellipses for the Incremental ED method is given in Table 2, and the best number of ellipses for the Intersecting ED method is given in Table 3.

3.2. *Second test — calculating the classifier accuracy*

After tuning the optimal number of ellipses in the previous section, the array set of the radius values, which was obtained through the optimizer, is stored for each dataset, and then, is

Table 1
Classification accuracy (%) for different number of ellipses on Gaussian ED in validation data.

Dataset	1	2	3	4	5	6	7	8	9	10
F-S	89.35	92.1	92	91.5	91.9	91.5	91.8	91.45	92.3	91.5
S–N	98.5	98.95	98.7	98.3	98.4	98.35	98.6	98.3	98.6	98.65
S–Z	97.1	97.7	98.1	98.15	98.1	98	97.95	98.05	98.05	95.97
S–ZNOF	98.5	98.48	98.4	98.44	98.34	98.48	98.42	98.36	98.38	98.48
A–B	44.5	43.1	46.4	44.43	43.2	42.6	42.8	43.5	44	43.7
A–C	87.1	87.4	87.4	87.3	87.4	88.4	88.10	87.6	87.7	87.4
B–C	99.6	100	100	99.8	99.8	99.9	100	100	99.8	99.8
AB–C	93.67	92.8	91.27	93.4	92.53	92.6	92.67	94.91	91.67	92.33
Average	88.54	88.82	**89.03**	88.92	88.71	88.73	88.79	89.02	88.81	88.48

Table 2
Best number of ellipses in Incremental ED.

Dataset	Optimal number of ellipses
F-S	4
S–N	4
S–Z	3
S–ZNOF	3
A–B	4
A–C	3
B–C	3
AB–C	3

Table 3
Best number of ellipses in Intersecting ED.

Dataset	Optimal number of ellipses
F-S	9
S–N	8
S–Z	7
S–ZNOF	9
A–B	5
A–C	10
B–C	7
AB–C	8

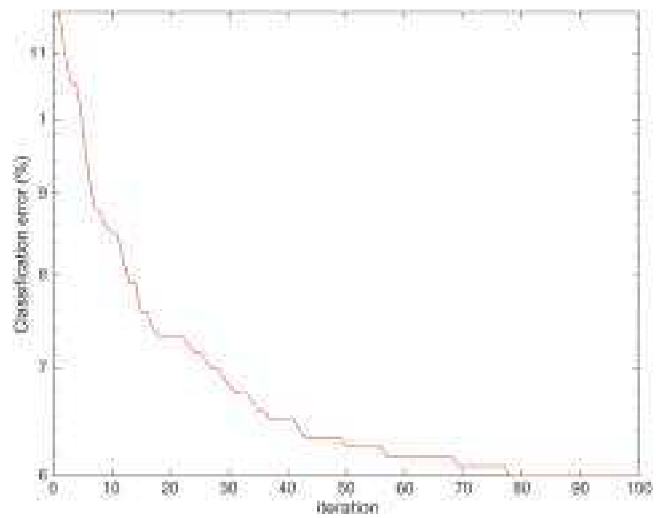

Fig. 9. Convergence graph in A–B dataet with In intersecting ED.

Table 4
Test accuracy of three method with 1-NN classifier.

Dataset	Gaussian ED acc (%)	Incremental ED acc (%)	Intersecting ED acc (%)
F-S	91.8	99	**100**
S–N	98.4	**100**	**100**
S–Z	97.7	**100**	**100**
S–ZNOF	98.28	99	**99.4**
A–B	46.1	**95**	94
A–C	87.4	98	**99**
B–C	**100**	**100**	**100**
AB–C	90.8	98.68	**99.33**
Average	88.81	98.71	**98.97**

applied to the testing data. The convergence graph of a sample database for the optimizer is shown in Fig. 9. The accuracy of the testing datasets for the three type of feature extraction techniques is given in Table 4. It can be seen that the Intersecting ED method is slightly superior to the Incremental ED, and both of these methods have very higher accuracy compared to the Gaussian ED. A sample of the phase space representation and the ellipses applied to the F-S dataset for three methods is shown in Fig. 10. As can be seen, Incremental ED method gives more options for choosing ellipse radiuses compared to Gaussian ED, and the intersection of the ellipses in Intersecting ED gives more complexity to the model and has a higher ability in the dataset classification.

3.3. Third test – a comparison with previous works

Finally, a comparison is made between the accuracy of the proposed algorithm and the works done by other researchers in the past. According to Table 5, for classification of the S–F dataset, the [46] has used the EMD and the adaptive noise methods, and reached the accuracy of 98% for separation of two classes (see Table 6).

In the cases of S–Z and S–N, several classification methods have reached an accuracy of 100%. For example, a classification method based on the complex network and edge weighting were discussed [47], which classified the epileptic seizures with an accuracy of 100% using the support vector machine classifier. In [48], the parts of the signal was broken down to subbands using the Tunable-Q Factor Wavelet Transform. This method has obtained the accuracy of 100% in classification. It should be noted that the proposed method of this study has obtained 100% accuracy in both databases.

In the two-class classification of the four types, S–ZNOF, the proposed method has higher accuracy compared to the previous works except for two cases. In the fourth classification, the proposed method was able to diagnose the two groups of epileptic and nonepileptic, at a reasonable rate. Even though the proposed method performs slightly weaker than the method of [49] and [50], low computational loads and high rate of the proposed method in obtaining the optimal response can be an advantage of the proposed method over MRBF-MPSO, GLCM+FV and SVM [49], and make it possible to use it in the real-time systems. Another better accuracy method, P-1D-CNN [50], has lower accuracy in other cases of classification. Also this method is a deep learning classifier, whereas proposed method is a feature extraction that can be fed to various classifiers.

In [51], the EEG signals are analyzed using the EMD and the adaptive noise methods; The different statistical features based on sub-signals are analyzed using the artificial neural network and obtained an accuracy of 98.87% in the classification. In [49],

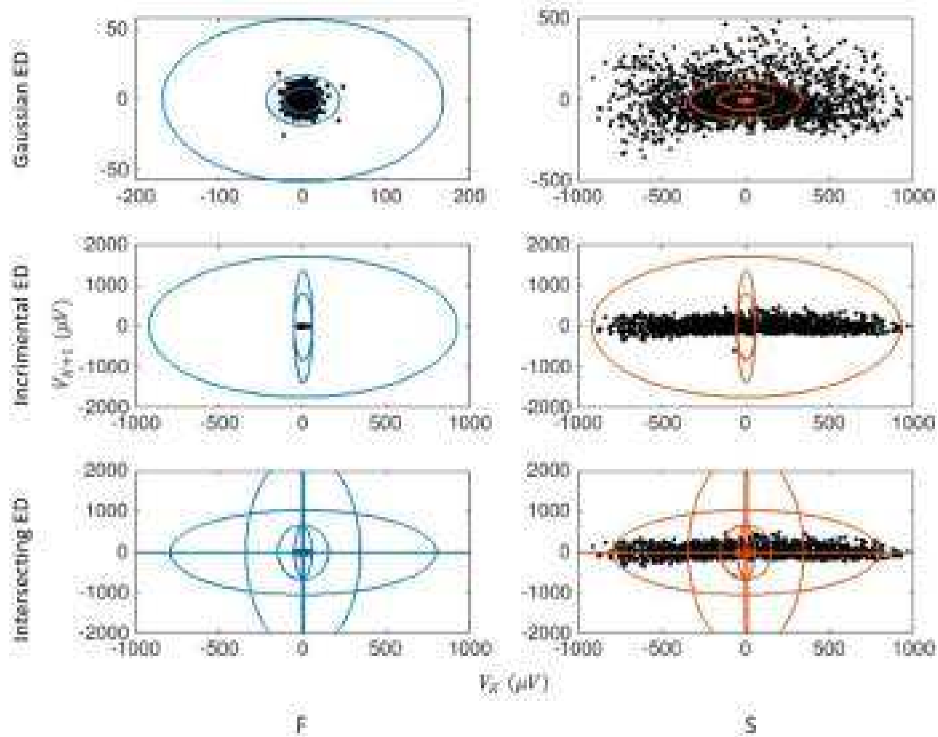

Fig. 10. Applying three methods in one F-S dataset sample.

an automatic classification method based on the multiscale radial basis function and the fisher vector encoding is presented. The accuracy obtained by this method is 99.53%.

For the second dataset, in [52], the classification accuracy of around 74.6%, 79.7%, 96.5%, and 91.8% was obtained for the A–B, A–C, B–C and A–B using the Hurst exponent value and the Autoregressive Moving Average parameters. In [49], the classifier has obtained the accuracies of 85.7%, 97.4%, 99.3%, and 98.2%, respectively. In comparison, the proposed method shows better classification performance with the results of 94%, 99%, 100% and 99.3%.

After the accuracy of the classifier, the second purpose of presenting this feature is to increase the speed of diagnosis for the EEG epileptic signals so that it can be used in the automatic real-time epilepsy diagnose systems. For this purpose, the calculation time of the mentioned features are compared with other feature extraction methods in Table 7. All simulations executed on a computer equipped with an Intel i7 processor and 8 Gb RAM. The timing are calculated using 'tic toc' command of MATLAB.

The features extracted from the wavelet transform consist of autoregressive model coefficients of order 4 [57], Shannon entropy values for the discrete wavelet packet transform at level 4 [58], an estimation of the second cumulant of the scaling exponents and the singularity spectrum. The mother wavelet used in the wavelet transform is the Daubechies2. This function has applications in biological signals.

The mentioned method of EMD is the extraction feature method used in [3], which consists of IMFs and transfer of which to the 2D phase space, and finally, calculation of the 95% confidence ellipse as the signal feature. Four IMFs was used for each signal.

The results indicate that the proposed method of Intersecting ED has a higher feature extraction rate compared to other methods. Although it does not has a high training rate, the test rate is very fast. The rates of the approximate entropy method and the empirical mode decomposition (EMD) method are very dependent on the lengths of the brain signal, and the rate of feature extraction significantly increases by increasing these lengths.

Table 5
Comparison of the proposed method with the existing methods studied on first dataset.

Dataset	Methods	Acc (%)
F-S	WPE and SVM [53]	96.5
	CEEMAN and LPBOOST classifier [54]	97
	CEEDMAN growth curve [46]	98
	MRBF-MPSO,GLCM+FV and SVM [49]	99.3
	P-1D-CNN, pyramid model M5 [50]	99.4
	Intersecting ED	**100**
S–N	WPE and SVM [53]	99.6
	DTCWT+GRNN [36]	**100**
	TQWT [48]	**100**
	MRBF-MPSO,GLCM+FV and SVM [49]	**100**
	P-1D-CNN, pyramid model M5 [50]	99.1
	Intersecting ED	**100**
S–Z	WPE and SVM [53]	99.5
	edge weight, LDA and SVM [47]	**100**
	ATFFWT+FD, LS-SVM [55]	**100**
	MRBF-MPSO,GLCM+FV and SVM [49]	**100**
	P-1D-CNN, pyramid model M5 [50]	**100**
	Intersecting ED	**100**
S–ZNOF	WPE and SVM [53]	97.38
	ATFFWT+FD, LS-SVM [55]	99.2
	CEEDMAN and ANN [51]	98.78
	MRBF-MPSO,GLCM+FV and SVM [49]	99.53
	MMSFL-OWFB based KE and SVM in 10-fold CV [56]	99.2
	P-1D-CNN, pyramid model M5 [50]	**99.7**
	Intersecting ED	99.4

4. Conclusion

In this study, a new method was presented for feature extraction on the classification of the EEG signals. In order to obtain

Table 6
Comparison of the proposed method with the existing methods studied on second dataset.

Dataset	Methods	Acc (%)
A–B	HURST, AMA [52]	74.6
	MRBF-MPSO,GLCM+FV and SVM [49]	85.7
	Intersecting ED	**94**
A–C	HURST, AMA [52]	79.7
	MRBF-MPSO,GLCM+FV and SVM [49]	97.4
	MMSFL-OWFB based KE and SVM in 10-fold CV [56]	98
	Intersecting ED	**99**
B–C	HURST, AMA [52]	96.5
	MRBF-MPSO,GLCM+FV and SVM [49]	99.3
	MMSFL-OWFB based KE and SVM in 10-fold CV [56]	**100**
	Intersecting ED	**100**
AB–C	HURST, AMA [52]	91.8
	MRBF-MPSO,GLCM+FV and SVM [49]	98.2
	MMSFL-OWFB based KE and SVM in 10-fold CV [56]	98
	Intersecting ED	**99.33**

Table 7
Comparison of the proposed method with the existing methods in mean speed computation..

Dataset	Methods	speed (s)
University of Bonn	ApEn [59]	9.54
	Wavelet Transform [60]	0.0481
	EMD [3]	0.7482
	Intersecting ED	**0.0061**
Neurology and Sleep Center of the Hauz Khas	ApEn [59]	0.905
	Wavelet Transform [60]	0.0131
	EMD [3]	0.0133
	Intersecting ED	**0.0012**

this method, first, the signals were mapped in a 2D phase space. Then, hypothetical ellipses were considered in the phase space representation. The number of points in each ellipse was given to the k-nearest neighbor classifier as a feature. In order to evaluate the efficiency of this feature, two different datasets were used. According to the results of the previous section, this feature extraction method has very high accuracy. Also, the rate of feature extraction and diagnosis of the epileptic signals is considerably high in this method compared to other conventional methods, therefore, this method can be implemented on the real-time epilepsy diagnosis systems, as method optimizes the radiuses by decreasing the misclassification rate in train data. Train data for real-time systems can be individual data from previous seizure attacks of mentioned person or general datasets of seizure attacks. Although the rate of learning in the training phase is not very high, a faster optimizer can be used to learn the radius values of the ellipses. Also, calculating the dimensions of the phase space such that the resulted figure of this space becomes a circle can be considered among the future works. By calculating the τ parameter, the central limit theorem characteristic is completely satisfied, and the distribution of the data becomes entirely gaussian. Therefore, the first feature extraction method, which is the Gaussian ED, can be used for classification and totally remove the learning phase from the optimization algorithms and increase the rate of the training phase of the classifier. The proposed method can be employed to EEG signal classification such as similar disorders, BCIs, sleep stages and simply, every single classifications with distinction of frequency and amplitude in each class signals. Finally, this feature extraction method can be used by other classifiers for various classification problems.

CRediT authorship contribution statement

Nastaran Darjani: Programmer, Software, Validation, Conceptualization, Visualization, Investigation, Writing - Reviewing and Editing, Writing - Original draft preparation. **Hesam Omranpour:** Supervision, Project administration, Conceptualization, Methodology, Visualization, Investigation, Writing - Reviewing and Editing, Funding acquisition, Programmer, Writing - Original draft preparation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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