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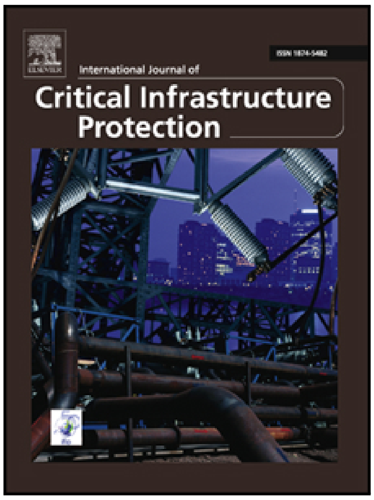

Modeling infrastructure interdependencies by integrating network and fuzzy set theory

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Modeling infrastructure interdependencies by integrating network and fuzzy set theory

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ABSTRACT

Infrastructure interdependency refers to the bidirectional relationship between entities, and means that the state of one entity is influenced by or correlated to the state of the other. Although some interdependencies in infrastructure networks can be modeled deterministically, often the required data are incomplete or there is an element of randomness in the relationships, necessitating the use of stochastic models. In this paper, the concepts and techniques of network and fuzzy set theory are integrated, and a fuzzy modeling approach is proposed to better identify and understand interdependencies and the relationships and connections between entities in infrastructure networks. This approach will allow the topological structures and characteristics of the network to be better understood for further evaluation and analysis.

Keywords: Infrastructure Network, Network Interdependency, Network Theory

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1. Introduction

An infrastructure network consists of the groups of interrelated entities that are essential to prosperity, security and life in society [14, 15]. Examples of infrastructure networks include power or energy generation networks, water supply networks, gas or oil supply networks, and logistics and supply chain networks. In these networks, entities are often highly interconnected and mutually dependent in complex ways. As such, a disruption to one entity can have a cascade effect to entities resulting in disruption to the entire network. For example, water supply networks generally include the entities of water storage facilities (e.g. reservoirs, water tanks, and water towers), water purification facilities (e.g. water and purification plants), and water pressurizing components (e.g. pumping stations and pumping gates). These entities are connected by water pipes, sewers, etc., so that the untreated water can be processed and then distributed to the consumers (which may be residential households or industrial or commercial establishments) as well as other usage points (such as fire hydrants). These entities in the water supply network are also related to other entities in other networks, such as the water pressurizing components which may be connected to entities in a power supply network, as they can assist in electricity generation. Similarly, the water supply network requires electricity to power it's facilities, so the water supply network also depends on the entities in the power supply network. Therefore, if a disruption occurs in one of the entities in the power supply network, this disruption may also affect the entities in the water supply network or even other infrastructure networks.

Models for these bidirectional relationships, or interdependencies, between entities have been considered in previous studies [28, 32]. As shown in Table 1, interdependencies can be classified according to the

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interactions, relationships, characteristics or effects between different types of entities or groups of entities in a network, and disruptive cascade effects and their magnitudes can also be anticipated based on the types of interdependencies between entities.

Table 1. Interdependency types [28, 32].

Interdependency Type	Description
Physical	Physical reliance on material flow from one infrastructure to another.
Cyber	Reliance on information transfer between infrastructures.
Geographic	Local environmental event affects components across multiple infrastructures due to physical proximity.
Logical	Dependency that exists between infrastructures that does not fall into one of the above categories.
Policy/ Procedural	Dependency that exists due to policies or procedures that relate the state of a component to a subsequent effect.
Societal	Influence that an infrastructure component event may have on societal factors.

Efforts have been made to model the different types of infrastructure interdependencies, based on the type of dependency and whether or not it is symmetric. For example, studies have considered models for physical interdependencies [8, 11, 16, 18, 23, 38, 39, 42, 43]; models for multiple-interdependencies, such as for physical and cyber interdependencies [5, 6, 10, 29, 30, 36], physical and geographic interdependencies [13, 31], physical, logical and societal interdependencies [17], physical, cyber and societal interdependencies [21, 34], and physical, cyber and geographical interdependencies [37].

Various approaches can be found in the literature for modeling these infrastructure interdependencies. For example, Hadjsaid et al. [10] adopt a cause and effect approach to model the relationships between power grids and information/communication systems in power and telecommunication networks. Svendsen and Wolthusen [36] use a scalable multigraph-based model with buffered resources to study the interdependencies among electric power, telecommunication and gas supply networks. Similarly, Wang et al. [39] apply a network model to analyze the interdependent responses under three types of edge disturbance strategies, and give a method for ranking critical components in the network. Beyeler et al. [3] use system dynamics models to identify chains of interdependencies arising from pervasive interconnections, which might create unexpected vulnerabilities or resilience. Most studies apply either analytical approaches to explicitly model infrastructure interdependencies [5, 7, 13, 16, 30, 34]; or use computer simulation techniques to simulate the infrastructure interdependencies under different situations for inference and prediction [3, 4, 10, 23, 37, 42].

1.1. Motivation

In the above literature, the majority of studies focus on models and maps for explicit interdependencies, e.g. physical and cyber interdependency. On the contrary, relatively few studies consider models of infrastructure interdependencies that may or may not exist. For example, the existence of physical interdependency can easily be verified by the existence or not of a physical linkage between infrastructures; however, the existence of a logical interdependency, policy/procedural interdependency, societal interdependency, etc., may be difficult to model and represent as such dependencies are difficult to identify and quantify. Moreover, the existence of some types of interdependency may be subjective depending on the views of different people or groups of people. The uncertainties in these interdependencies also arise from incomplete/inaccurate interpretation about the states or the randomness in the type of interdependency.

Insufficient data may also lead to uncertainties in the modeling of the infrastructure interdependencies. Access to the necessary data is often difficult because the vast majority of infrastructure is owned by the private sector and there are significant barriers to sharing information with the public sector [33]. Furthermore, experts from a particular sector may only be able to identify the interdependencies within that sector, and may lack the expertise to identify dependencies with other types of infrastructure.

Thus, a major challenge is to model and identify both the deterministic and stochastic interdependencies between different sectors, so as to encapsulate the topological structure of infrastructure networks for further analysis and evaluation and for disruption prevention, protection and recovery; and also for the development of more resilient infrastructure networks.

1.2. Contributions

To model complex network interdependencies, some studies consider a fuzzy modeling approach. For example, Stergiopoulos et al. [35] approximate the time evolution of a cascading failure using fuzzy approximations of impact evolution; Oliva et al. [26] adopt fuzzy measures to develop criticality indices to rank the physical interdependencies in an infrastructure network; Yazdani et al. [41] adopt a fuzzy multi criteria decision-making technique to determine the weights and the importance of alternatives with respect to infrastructure interdependencies; Akgun et al. [1] present a fuzzy cognitive map methodology to determine the vulnerabilities of interdependent infrastructures under multiple criteria; and Muller [20] presents a fuzzy modeling method to forecast the impacts of a disruptive event and the resilience of an infrastructure network.

This paper presents an extension of the fuzzy modeling approach. Previous studies assume that the interdependencies in a network are known in advance, and are not able to fully reproduce phenomena such as cascades. A fuzzy modeling approach can be applied either to assess the performance of the explicit interdependencies so as to reduce the ambiguity of available alternatives [1, 12, 26, 27, 41] or to forecast the disruption due to cascades [20, 25, 34, 35].

The approach proposed in this paper integrates network and fuzzy set theory to model not only the explicit or physical interdependencies, but also to model and investigate uncertain or hidden (non-physical) interdependencies. The proposed approach can retrospectively describe and reveal these uncertain or hidden (non-physical) interdependencies, helping decision makers to anticipate and prevent potential disruption events and cascade effects, by modeling the infrastructure interdependencies (both the explicit and uncertain interdependencies) with nodes.

The proposed modeling approach can also infer the topology of infrastructure networks from disruptions. This means that disruption prevention strategies or recovery planning of infrastructure networks can be designed based on the simulated disruption events. In the proposed approach, the network topology can be varied to maximize resilience against disruption from both explicit and hidden dependencies.

The proposed modeling approach provides topological insight to represent infrastructure interdependencies as well as an appropriate approach to handle the uncertainties of these interdependencies under situations that are crisp and non-deterministic, and cannot be described precisely or if the complete description of the situations requires more data. The proposed fuzzy modeling approach is then able to identify and model the significant interdependencies that exist in an infrastructure network by considering the connectivity and relationships of the entities in the network.

This paper provides new insight and a methodology for the identification of infrastructure interdependencies and the modeling of infrastructure interdependencies under the cascade effect, in which both the explicit and uncertain interdependencies can be modeled simultaneously.

2. Fuzzy Modeling Approach for Infrastructure Interdependency

In this paper, a fuzzy modeling approach is proposed for the inference of infrastructure interdependencies. The proposed approach integrates network theory and fuzzy set theory to model interdependencies. Network theory is adopted to build the network, and fuzzy set theory is adopted to determine the degree of interdependency between the connected infrastructures in the network. The fuzzy concepts and techniques provide an appropriate approach for situations that are too complex to model deterministically [44]. Unlike existing approaches [1, 12, 20, 25, 26, 27, 34, 35, 41], the proposed integrated approach also considers situations in which one entity may have multiple connections with other entities, and thus can identify potential interdependency between infrastructures as well as the importance of the interdependency.

The proposed approach first assumes that the network is a complete graph network, and that every entity is connected to every other entity in the network, such that every pair of entities is connected by a unique link. Then, the complete graph for the infrastructure network with n entities is denoted by k_n, where k_n has $\frac{n(n-1)}{2}$ links with its own maximal cliques. In the integration of fuzzy set theory with the complete graph for an infrastructure network, it is assumed that k_n has a set of infrastructures, and each of these infrastructures has a relationship with other infrastructures via a link l_{ij} for $n_i, n_j \in k_n$. The link l_{ij} denotes that infrastructure n_i is dependent on n_j and infrastructure n_j is dependent upon n_i, and the link may be reflexive in some situations, i.e. $l_{ij} \leftrightarrow l_{ji}$. Therefore, an infrastructure network k_n can be represented by a set of nodes (infrastructures) and links, $k_n = \{N, L\}$, where $N = \{n_1, n_2, \dots, n_i, n_j, \dots\}$ is the set of nodes (infrastructures) in the network k_n, and $L = \{l_{12}, l_{23}, \dots, l_{ij}, \dots\}$ is the set of interdependencies over the links in the network. From the modeling of the fuzzy variables, the degree of the nodes and the interdependency over a link can be easily associated with the corresponding nodes in the network. Moreover, the fuzziness of the interdependency between the corresponding nodes can also be identified by fuzzy set theory. The relationships between the fuzzy variables are illustrated in Fig. 1.

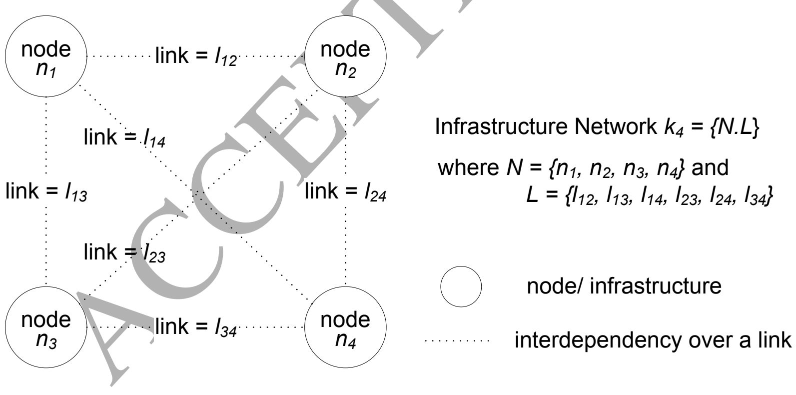


Figure 1. Relationships between the fuzzy variables.

Since the existence of interdependencies are not always known in advance, the linguistic variables of fuzzy set theory are proposed to minimize the effect of noisy data. Here, let S be a set of linguistic variables that corresponds to L, and the set of linguistic variables $S = s_{12}, s_{23}, \dots s_{ij}, \dots$ has the same properties as L. Furthermore, each of the linguistic terms in the set is characterized by a fuzzy set F_{ik} with membership function $\mu_{F_{ik}}$, so the value of the linguistics variable s_{ik} is represented as $s_{ik}(l_{ik})$ with the degree of membership denoted as $\mu_{F_{ik}}(l_{ik})$. This representation of networks as nodes is useful because a node can have more than one interdependency with other nodes in a network, including stochastic interdependencies.

2.1. Formulating the interdependencies

In infrastructure networks, it is assumed that interdependencies are formed among the infrastructures as nodes, and each node is assumed to be connected with all other nodes in the infrastructure network, such that each node should have at least one connection to another node in the network, i.e. if n_j depends on another n_i, for $n_i, n_j \in k_n$ and $n_i \neq n_j$, it is then expected that an interdependency is formed between n_i and n_j, and the degree of occurrence of the interdependency can then be modeled by fuzzy set theory. In fuzzy dependency, the fuzzy connection between n_i and n_j, $s_{ij}(l_{ij})$, is the fuzzy interdependency between n_i and n_j in terms of s_{ij} . This fuzzy interdependency refers to the first order interdependency between a pair of nodes; a primary and single interdependency with another node in the infrastructure network. Then the degree $d\left(s_{ij}(l_{ij})\right)$ of the occurrences of the fuzzy interdependency for the nodes in the infrastructure network is defined as

$$d\left(s_{ij}(l_{ij})\right) = \sum_{i=1}^{j} \min \left(\mu_{F_j}(n_j), \mu_{F_i}(n_i)\right). \tag{1}$$

Since the link l_{ij} can be a directional or unidirectional interdependency between node n_i and node n_j, a minimization function is applied in Equation (1) to avoid the duplication of membership functions for the fuzzy interdependencies. To determine the significance of the fuzzy interdependency, the degree of the occurrences of the fuzzy interdependency $d\left(s_{ij}(l_{ij})\right)$ is compared to the expected occurrences of the fuzzy interdependency $e\left(s_{ij}(l_{ij})\right)$ based on the assumption that each node is connected with all other nodes in the infrastructure network. Then according to the standardized residual approach [9], the scaled difference is determined as

$$std_{ij} = \frac{e\left(s_{ij}(l_{ij})\right) - d\left(s_{ij}(l_{ij})\right)}{\sqrt{e\left(s_{ij}(l_{ij}) \to s_i(n_i)\right)}}, \tag{2}$$

where $e\left(s_{ij}(l_{ij})\right) = \sum_{i=1}^{|N|} \mu_{F_i}(n_i) \times \sum_{j=1}^{|N|} \mu_{F_j}(n_j) \times \sum_{i=1}^{|N|} \sum_{j=1}^{|N|} \min\left(\mu_{F_j}(n_j), \mu_{F_i}(n_i)\right).$ (3)

The standardized residual approach measures the distance between the degree of the occurrences and the expected occurrence of the fuzzy interdependency. The expected occurrences of the fuzzy interdependency refers to the possible number of interdependencies in an infrastructure network, which means the maximum number of edges that a node can have in order to form interdependencies with other nodes, i.e. the unions of the sets of the nodes with interdependencies in the network. The minimization function is also applied in the determination of the expected occurrences of the fuzzy interdependency to avoid the duplication of the membership functions of the fuzzy interdependencies.

The standardized residual approach approximates a standard normal distribution; however, the number of infrastructure interdependencies may not follow a normal distribution, so an adjustment [2, 40] is applied to the standardized residual std_{ij} to give the adjusted residual adj_{ij},

$$adj_{ij} = \frac{std_{ij}}{\sqrt{est_{ij}}}, \tag{4}$$

where est_{ij} is the maximum estimation for the asymptotic variance [9], and is defined as

$$est_{ij} = \left(1 - \sum_{i=1}^{|N|} \mu_{F_i}(n_i)\right) \times \left(1 - \sum_{j=1}^{|N|} \mu_{F_j}(n_j)\right). \quad (5)$$

The maximum estimation for the asymptotic variance is used to derive an approximate probability distribution for the membership function of the fuzzy interdependencies that is asymptotically normal. Therefore, from the adjusted residual adj_{ij}, the fuzzy interdependency $s_{ij}(l_{ij})$ is significant when the value of $\mathrm{adj_{ij}}$ is within the 95% confidence level, meaning that $\mathrm{n_i}$ is dependent on $\mathrm{n_j}$.

Since a node can have multiple interdependencies or connections with other nodes in a network, the fuzzy interdependency can be characterized by more than one fuzzy term. Multiple interdependencies are also called higher order interdependencies (e.g. second order interdependency, third order interdependency) of a node. Thus, the *nth* order interdependencies are constructed from the lower order $(n-1)th$ interdependencies of a node. For example, the second-order interdependency is based on the first-order interdependency of node n_i with its two dependent nodes n_j and node n_k, i.e., the fuzzy interdependencies $s_{ij}(l_{ij})$ and $s_{ik}(l_{ik})$. Similarly, the third-order interdependency is based on the second-order interdependency of node n_i with its three dependent nodes n_j, n_k and n_l, and the fuzzy interdependencies $s_{ij}(l_{ij})$, $s_{ik}(l_{ik})$ and $s_{il}(l_{il})$. By iteratively determining the orders of the fuzzy interdependencies, the significance of the interdependencies of the nodes in the infrastructure networks can then be modeled.

2.2. Weighting the fuzzy interdependencies

The existence, or exact nature of infrastructure interdependencies are not necessarily known in advance, so the uncertainty associated with $s_{ij}(l_{ij})$ is defined probabilistically as $P\left(s_{ij}(l_{ij})|s_{(i-1)(j-1)}(l_{(i-1)(j-1)})\right)$. Therefore, to predict $s_{ij}(l_{ij})$, a weight of evidence measure, $W\left(s_{ij}(l_{ij})\right)$ [24], is adopted to define the mutual information for the set of fuzzy interdependencies,

$$W\left(s_{ij}(l_{ij})\right) = \log\frac{P\left(s_{ij}(l_{ij})|s_{(i-1)(j-1)}(l_{(i-1)(j-1)})\right)}{Pr\left(s_{ij}(l_{ij})\right)} - \log\frac{P\left(s_{ij'}(l_{ij'})|s_{(i-1)(j-1)}(l_{(i-1)(j-1)})\right)}{Pr\left(s_{ij}(l_{ij})\right)}, \quad (6)$$

where $P\left(s_{ij}(n_{ij})|s_{(i-1)(j-1)}(l_{(i-1)(j-1)})\right)$ is the probability of the fuzzy interdependency between n_i and n_j, i.e. $s_{ij}(l_{ij})$ given $s_{(i-1)(j-1)}(l_{(i-1)(j-1)})$, and $P\left(s_{ij'}(l_{ij'})|s_{(i-1)(j-1)}(l_{(i-1)(j-1)})\right)$ is the probability of the fuzzy interdependency between n_i and nodes other than n_j, i.e. $s_{ij'}(l_{ij'})$ given by $s_{(i-1)(j-1)}(l_{(i-1)(j-1)})$. These conditional probabilities are applied to determine the probability of a fuzzy interdependency existing between nodes. Therefore, the weight of evidence measure $W\left(s_{ij}(l_{ij})\right)$ measures the significance of the fuzzy interdependency between n_i and its dependent nodes, i.e. for accepting or rejecting the existence of $s_{ij}(l_{ij})$. Therefore, the triangular fuzzy numbers, F, for representing the pair-wise weightings and the interdependency are defined as the set $F = \{(x, \mu_{Fx}), x \in R\}$, where $-\infty < x < +\infty$, and μ_{Fx} is a continuous mapping from R to the closed interval $[0, 1]$. Moreover, the states of the interdependency are defined as "highly interdependent (U)", "averagely interdependent (M)", and "low interdependent (L)" as shown in Fig. 2. Then, a triangular fuzzy number denoted as $\widetilde{M} = (a, b, c)$ for $a \le b \le c$ under the triangular-type membership function is defined as follows:

$$\mu_{\widetilde{M}}(x) = \begin{cases} 0 & for\ x < a \\ (x-a)/(b-a) & for\ a \le x \le b \\ (c-x)/(c-b) & for\ b \le x \le c \\ 0 & for\ x > c \end{cases}. \quad (7)$$

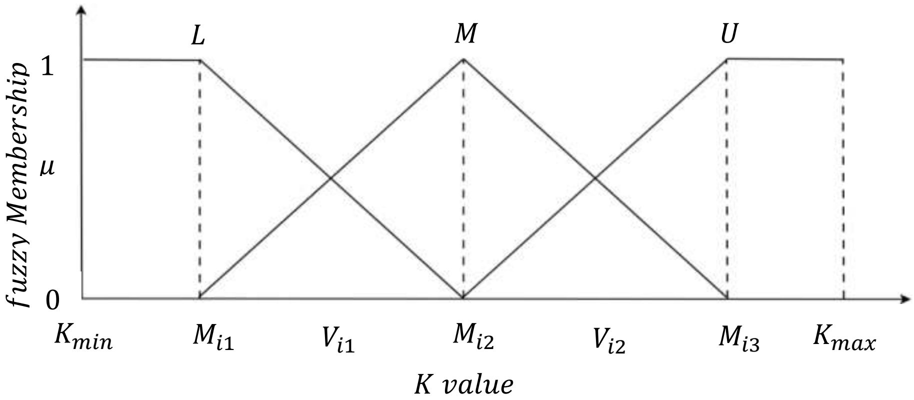


Figure 2. Membership functions.

The fuzzy interdependency between n_i and n_j is shown in Fig. 2, where K_{max} and K_{min} are the maximum and minimum values for an interdependency to be formed between the nodes in the network, and V_{i1} (V_{i2}) represents the threshold for the lower (upper) tercile of the measurements. Following Mitra and Acharya [19], V_{i1} and V_{i2} are determined by dividing the region into intervals of equal width (δ), and then obtaining the corresponding class frequencies, so the position of the k-th partition is defined as

$$V_{ik} = low_i + \frac{R_k - cf_{i-1}}{f_i} \times \delta, \tag{8}$$

where low_i is the lower limit of the i-th class interval, R_k is the rank of the k-th partition value, and cf_{i-1} is the cumulative frequency of the immediately preceding class interval such that $cf_{i-1} < R_k < cf_i$. Therefore, the values of M_{i1}, M_{i2} and M_{i3} in the membership function are $M_{i1} = \frac{K_{min}+P_{i1}}{2}$, $M_{i2} = \frac{P_{i1}+P_{i2}}{2}$, and $M_{i3} = \frac{P_{i2}+K_{max}}{2}$. Then, the degree of membership for the states of the interdependency can be represented as

$$\mu_L(n_i) = \begin{cases} 1 & for\ n_i \le M_{i1} \\ \frac{M_{i2}-n_i}{M_{i2}-M_{i1}} & for\ M_{i1} < n_i < M_{i2}, \\ 0 & otherwise \end{cases} \tag{9}$$

$$\mu_M(n_i) = \begin{cases} 0 & for\ n_i < M_{i1} \\ \frac{n_i-M_{i1}}{M_{i2}-M_{i1}} & for\ M_{i1} \le n_i \le M_{i2} \\ \frac{M_{i3}-n_i}{M_{i3}-M_{i2}} & for\ M_{i2} \le n_i \le M_{i3} \\ 0 & for\ n_i > M_{i3} \end{cases}, \tag{10}$$

$$\mu_U(n_i) = \begin{cases} 1 & for\ n_i \le M_{i2} \\ \frac{n_i-M_{i2}}{M_{i3}-M_{i2}} & for\ M_{i2} < n_i < M_{i3}. \\ 0 & otherwise \end{cases} \tag{11}$$

Therefore, the above proposed fuzzy modeling approach can be used to model the infrastructure interdependencies that are not explicitly connecting nodes, or the interdependencies that are not completely deterministic in the infrastructure network.

3. Examples

This section presents illustrative examples of the proposed fuzzy modeling approach for modeling infrastructure interdependency and developing a more resilient infrastructure network.

3.1. *Modeling the infrastructure interdependencies with nodes*

An example of a complete graph, k_7, for a network consisting of 7 infrastructures with 7 nodes and 21 links is illustrated in Fig. 3. Among the 21 links, some interdependencies may be explicit links to other nodes while other interdependencies may be implicit or stochastic links. The computed values for the adjusted residuals and weights for the fuzzy interdependencies $s_{12}(l_{12}), \cdots, s_{ij}(l_{ij}), \cdots, s_{67}(l_{67})$ in the complete graph k_7 are presented in Tables 2 and 3, respectively. The topology of the infrastructure network with the degree of occurrence of the fuzzy interdependencies is shown in Fig. 4.

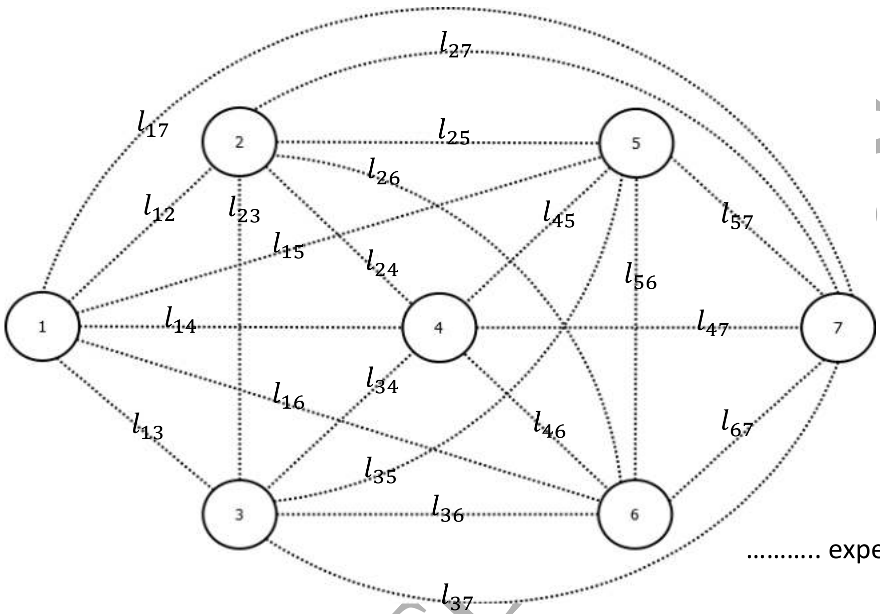


Figure 3. Complete graph k_7 with the maximum expected occurrences of infrastructure interdependency.

Table 2. Values of the adjusted residuals (asymptotic variance = 1) for the fuzzy interdependencies.

	Adjusted Residuals						
From \ To	n_1	n_2	n_3	n_4	n_5	n_6	n_7
n_1	-	0.88	1.00	0.14	0.14	0.14	0.14
n_2	0.88	-	1.00	0.22	0.85	0.28	0.14
n_3	1.00	1.00	-	1.00	0.33	0.33	0.28
n_4	0.14	0.22	1.00	-	1.00	0.83	0.28
n_5	0.14	0.85	0.33	1.00	-	1.00	0.91
n_6	0.14	0.28	0.33	0.83	1.00	-	1.00
n_7	0.14	0.14	0.28	0.28	0.91	1.00	-

Table 3. Weights of the expected occurrences of the fuzzy interdependencies.

To / From	Expected Occurrence n_1	n_2	n_3	n_4	n_5	n_6	n_7
n_1	-	0.77	1	0.02	0.02	0.02	0.02
n_2	0.77	-	1	0.05	0.72	0.08	0.02
n_3	1	1	-	1	0.11	0.11	0.08
n_4	0.02	0.05	1	-	1	0.69	0.08
n_5	0.02	0.72	0.11	1	-	1	0.82
n_6	0.02	0.08	0.11	0.69	1	-	1
n_7	0.02	0.02	0.08	0.08	0.82	1	-

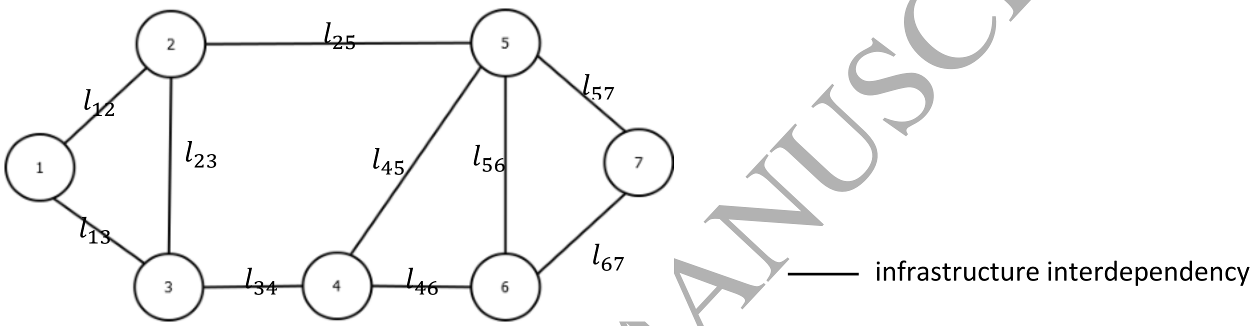


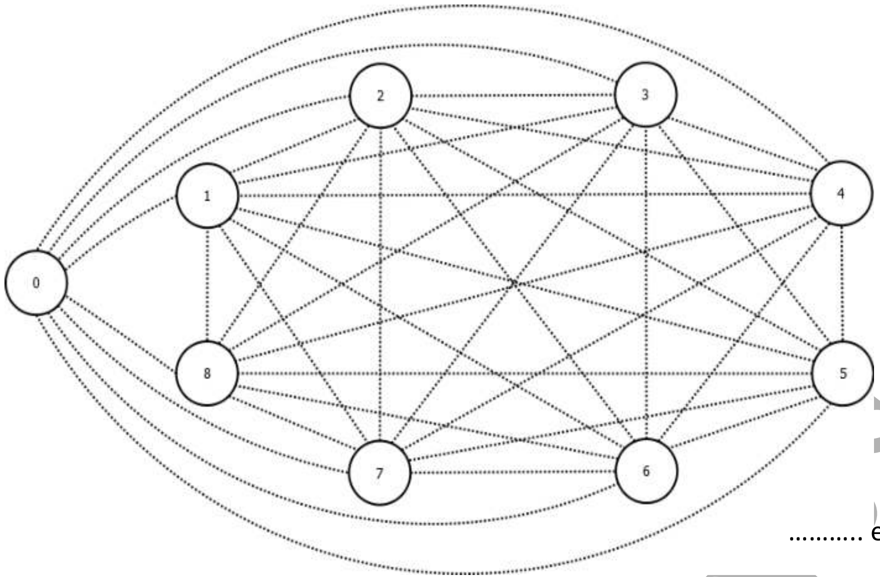
Figure 4. Topological structure of an infrastructure network with identified interdependencies.

According to the proposed fuzzy modeling approach, the network in Fig. 4 has 10 significant interdependencies among the infrastructures, in which the degrees of the occurrences of the interdependencies are less than the expected occurrences of the interdependencies as shown in the corresponding complete graph k_7 in Fig. 3. The proposed approach thus can be used to help model the interdependencies between entities in a network, by revealing previously unknown relationships between entities.

3.2. Modeling the infrastructure interdependencies from a disruption

The proposed approach can also be used to model the infrastructure interdependencies from a disruption, i.e. the interdependencies induced by the cascade effect of a disruption. The proposed approach is illustrated with simulated data from McDaniels et al. [16]. The data used in [16] are intended for the development of a framework to characterize infrastructure failure interdependencies in infrastructure networks, in which the sources of data and information include news in printed media sources and technical reports prepared by the responsible agencies.

Here, we extend the results of McDaniels et al. [16] to consider the cascade effect from a power network disruption to other sectors, such as agriculture and food production, banking and finance, communication and information technology, drinking water and treatment plants, military installations and defense, health care and civil services, transportation systems, and commercial and industrial services. The data used in this illustrative example are obtained according to the approach proposed by Nojoma and Kameda [22] for interactions stemming from a specific extreme event. The data used in this illustrative example came from similar sources to those used in Nojoma and Kameda [22], including media reports and the official reports from responsible agencies. Then, to model the infrastructure interdependencies between the disruption event and the eight interrelated sectors listed above, the complete graph k_9 with 9 nodes and 36 links is shown in Fig. 5, and the estimated adjusted residuals and weights for the fuzzy interdependencies $s_{12}(l_{12}), \cdots, s_{ij}(l_{ij}), \cdots, s_{89}(l_{89})$ are presented in Tables 4 and 5, respectively.



0: A Disruption
1: Agriculture and food production
2: Banking and finance
3: Communication and information technology
4: Drinking water and treatment plants
5: Military installations and defense
6: Health care and civil services
7: Transportation systems
8: Commercial and industrial services

……….. expected infrastructure interdependency

Figure 5. Maximum expected occurrences of the infrastructure interdependencies from a disruption.

Table 4. Values of the adjusted residual for the interdependencies.

Adjusted Residuals									
From \ To	n_0	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8
n_0	-	0.75	0.83	0.87	0.72	0.71	0.78	0.88	0.87
n_1	0.75	-	0.14	0.14	0.36	0.69	0.46	0.79	0.78
n_2	0.83	0.14	-	0.91	0.17	0.88	0.35	0.47	0.83
n_3	0.87	0.14	0.91	-	0.22	0.91	0.65	0.76	0.81
n_4	0.72	0.36	0.17	0.22	-	0.69	0.46	0.44	0.41
n_5	0.71	0.69	0.88	0.91	0.69	-	0.65	0.78	0.72
n_6	0.78	0.46	0.35	0.65	0.46	0.65	-	0.46	0.33
n_7	0.88	0.79	0.47	0.76	0.44	0.78	0.46	-	0.78
n_8	0.87	0.78	0.83	0.81	0.41	0.72	0.33	0.78	-

Table 5. Values of the expected occurrences of the fuzzy interdependencies.

Expected Occurrence of the Fuzzy Interdependency									
From \ To	n_0	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8
n_0	-	0.56	0.69	0.76	0.52	0.51	0.61	0.77	0.75
n_1	0.56	-	0.02	0.02	0.13	0.48	0.21	0.62	0.61
n_2	0.69	0.02	-	0.82	0.03	0.77	0.12	0.22	0.69
n_3	0.76	0.02	0.82	-	0.05	0.82	0.42	0.58	0.66
n_4	0.52	0.13	0.03	0.05	-	0.48	0.21	0.19	0.17
n_5	0.51	0.48	0.77	0.82	0.48	-	0.42	0.61	0.52
n_6	0.61	0.21	0.12	0.42	0.21	0.42	-	0.21	0.11
n_7	0.77	0.62	0.22	0.58	0.19	0.61	0.21	-	0.61
n_8	0.75	0.61	0.69	0.66	0.17	0.52	0.11	0.61	-

The computed values of the degree of occurrence of the fuzzy interdependencies are shown in Figs. 6 and 7. A total of 23 significant infrastructure interdependencies can be found between the disruption event (node n_0) and the 8 identified areas (node n_1 to node n_8). The 23 significant infrastructure interdependencies show the cascade effect of a disruption.

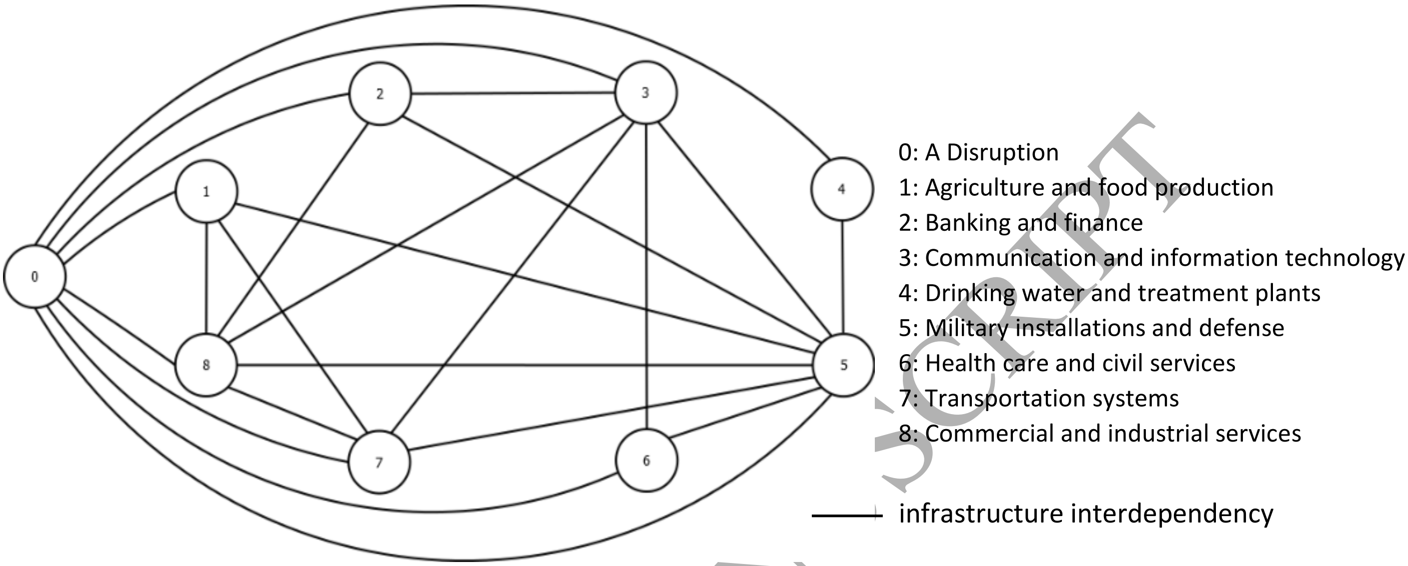


Figure 6. Topological structure with the identified interdependencies from a disruption.

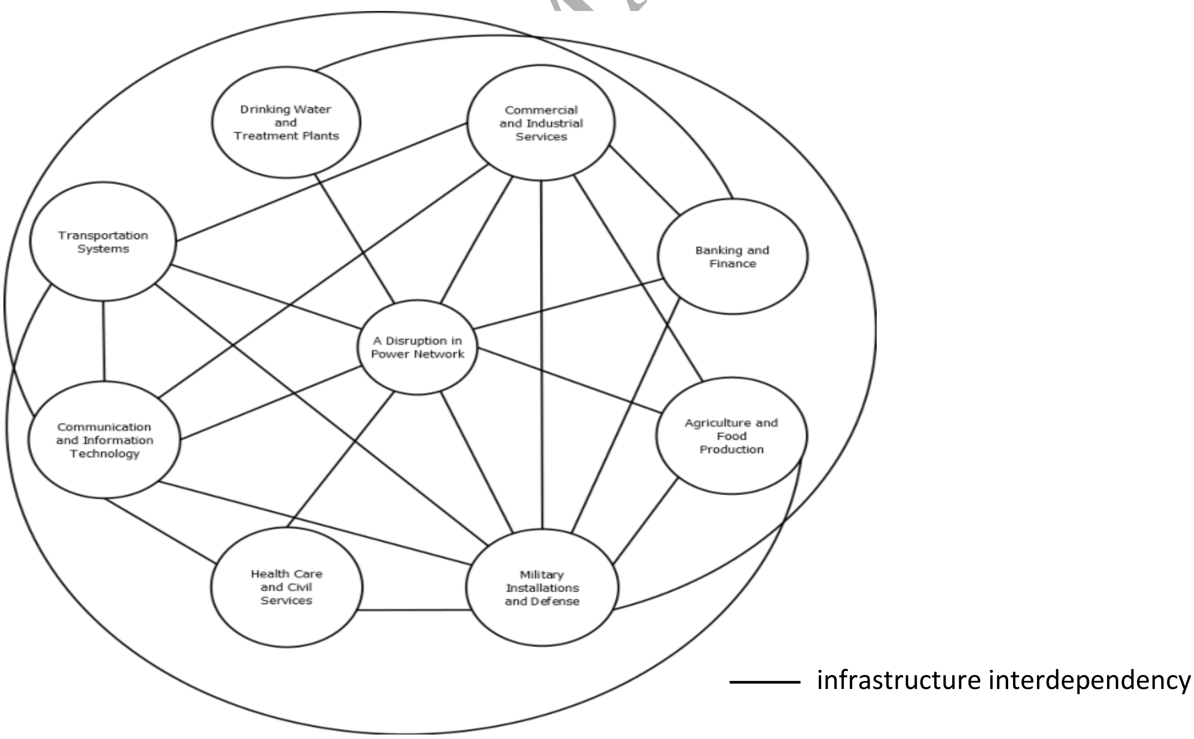


Figure 7. Topological structure with identified interdependencies resulting from a power network disruption.

Comparing the results in Fig. 7 in this paper with Fig. 8 in McDaniels et al. [16] (which considers the infrastructure failure interdependencies and the consequences for the 2003 northeast blackout), the proposed fuzzy modeling approach can produce very similar results for modeling the infrastructure interdependencies from a power network disruption. As shown in Table 6, the proposed approach also covers food supply, finance, telecommunications, utilities, etc., as in the work of McDaniels et al. [16]. Moreover, besides modeling the infrastructure interdependencies from a disruption, the proposed approach can also help to identify previously unknown interdependencies. This is useful for increasing network resilience.

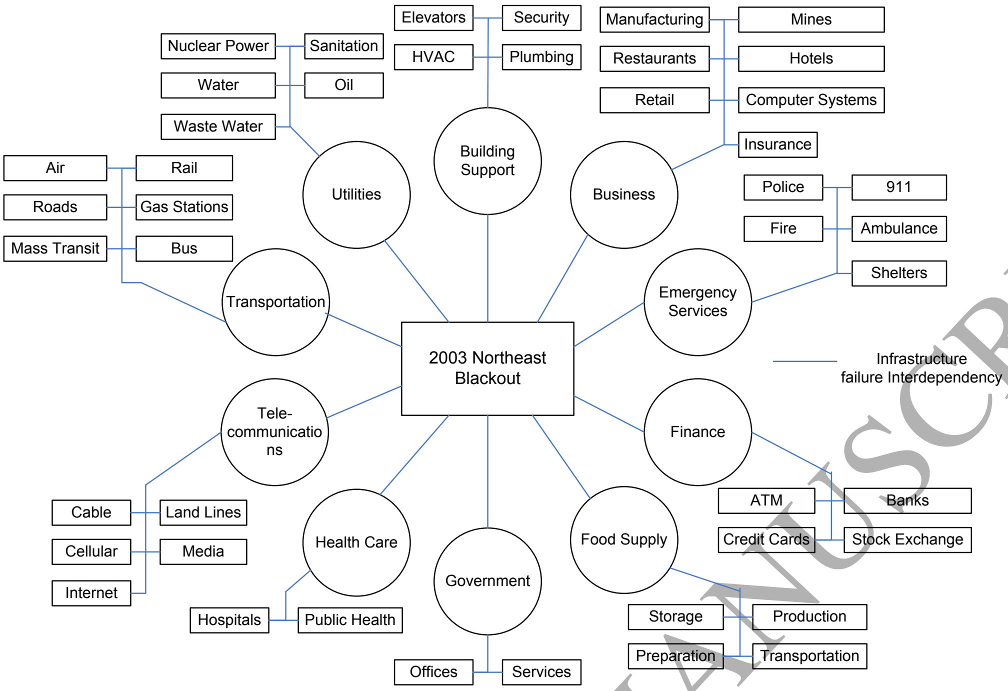


Figure 8. Infrastructure failure interdependencies and their consequences for the 2003 northeast blackout (McDaniels et al. [16]).

Table 6. Comparison between the identified interdependencies in a power network disruption event.

	Proposed Approach	McDaniels et al. [16]
Identified infrastructure interdependencies between network/entities in a power network disruption	Agriculture and Food production	Food Supply
	Banking and Finance	Finance
	Communication and Information Technology	Telecommunications
	Drinking Water and Treatment Plants	Utilities
	Military Installations and Defense	Government
	Health Care and Civil Services	Health Care Emergency Services Building Supports
	Transportation Systems	Transportation
	Commercial and Industrial Services	Business

4. Conclusion

In this paper, a fuzzy modeling approach was proposed to integrate network and fuzzy set theory to model infrastructure interdependency. Network theory provides topological insight to help represent the infrastructure interdependencies, while fuzzy set theory provides a suitable approach to handle uncertainties in infrastructure interdependencies under situations that are crisp and non-deterministic. The proposed fuzzy modeling approach can be used to determine which interdependencies are statistically significantly, and to model these interdependencies in an infrastructure network by considering the connectivity and relationships of the entities in the network.

Some interdependencies in a network may not be identifiable until a disruption occurs. However, these independencies may have potential implications for the network in terms of the resilience, robustness, or industrial/commercial values. The proposed approach can also be applied to model these interdependencies between entities by simulating the effects of a disruption. The cascade effect from a disruption in a network can be difficult to model deterministically, but the proposed approach can help to reveal the interdependencies arising from the cascade effect. The modeled interdependencies and connections between the entities that are obtained with the proposed approach (e.g. the order of interdependency and the number of paths between a pair of entities) may also be used to further analyze and evaluate the topological structures of the network as well as the resilience of the network. Therefore, the proposed fuzzy modeling approach may help to better infer and reveal the interdependencies in infrastructure networks, and assist in reconstructing networks and increasing their resilience. An area for future work is how to model and consider directed and multiple interdependencies between infrastructures. Additionally, the space, distance or capacity should be considered in the modeling of infrastructure interdependencies. By having a more detailed representation of the topology of an infrastructure network, further analysis and evaluation on how the network is influenced by an entity or clusters of entities can be conducted, allowing more resilient infrastructures to be developed.

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